Optimal Competitive Online Ray Search with an Error-Prone Robot

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The Problem

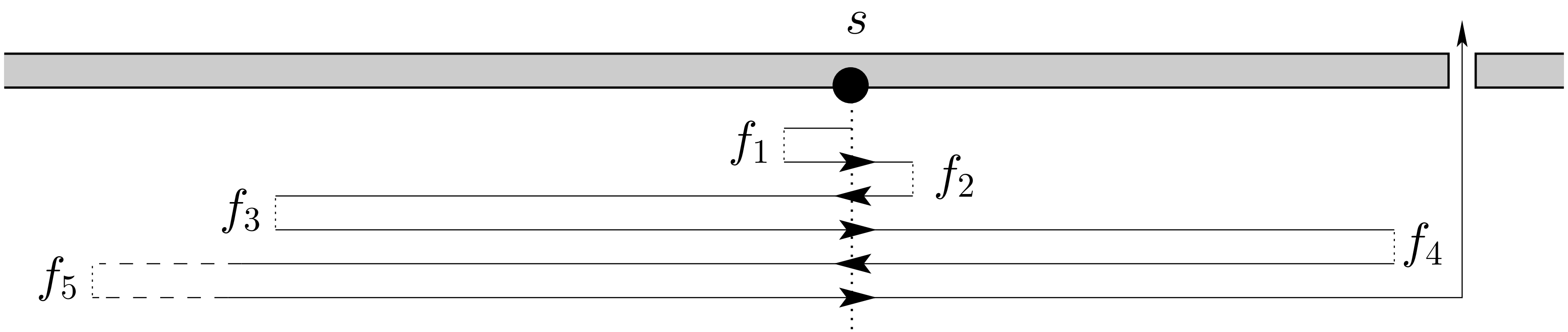
Robot

- No Vision
- Located in front of a wall
- Direction to door unknown
- Distance to door unknown

Task

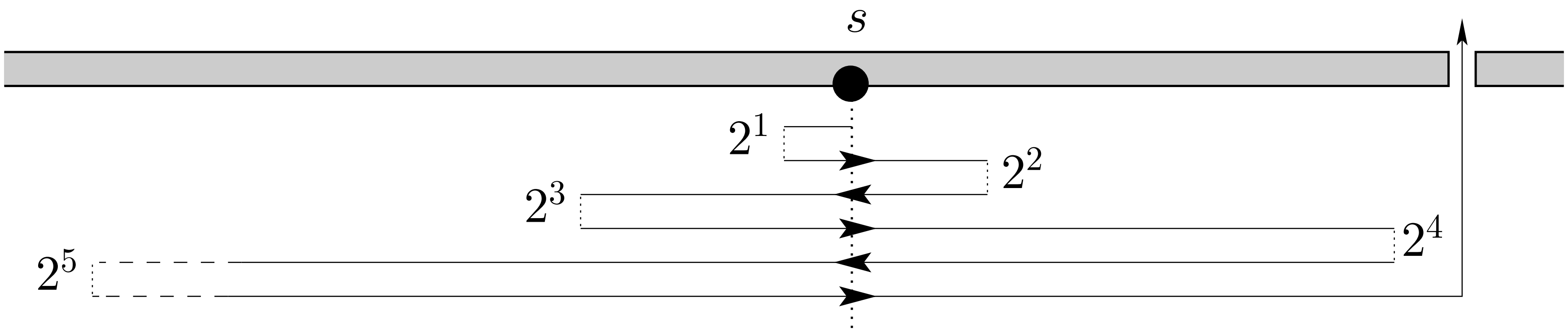
- Find the door

Solving this Problem



- *Strategy:* sequence $F = (f_i)_{i \in \mathbb{N}}$, $f_i > 0$
- Walk f_i steps $\begin{cases} \text{to the left} & \text{if } i \text{ is odd} \\ \text{to the right} & \text{if } i \text{ is even} \end{cases}$ and back to s

The Doubling Strategy



- *doubling* strategy: $f_i = 2^i$
- Competitive factor: $\dfrac{\text{Distance covered by the robot}}{\text{Shortest path}} \leq 9$
- 9 is optimal [*Baeza-Yates et. al., Gal*]

Problem

"Go one unit"

Idealistic Robot

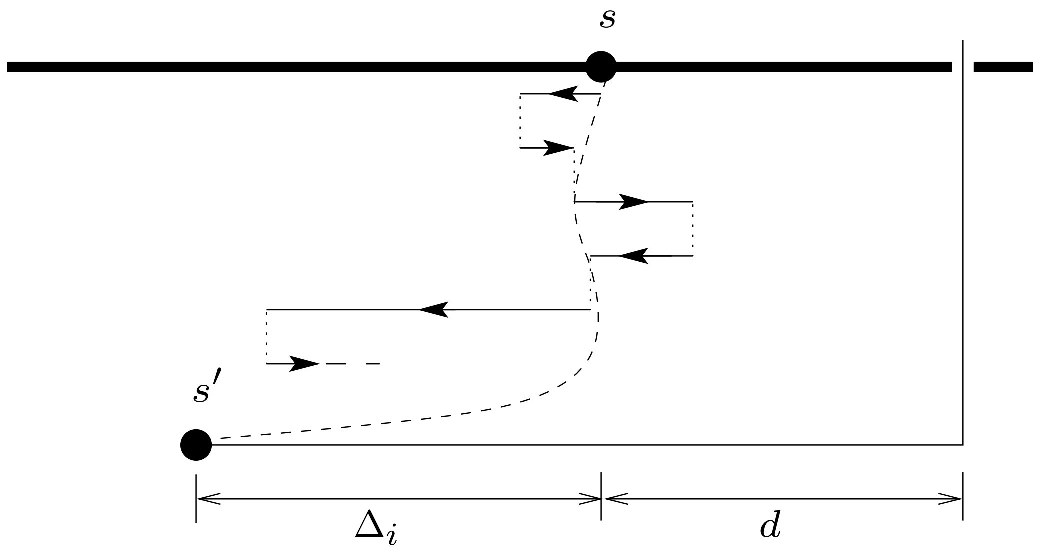
1

Real Robot

$1 - \delta$ $1 + \delta$

Error bound by $\delta \in [0, 1[$

Drift



- Problem: Robot may not return to the start point (*drift*)

Standard Doubling Strategy

Standard doubling strategy $f_i = 2^i$ with error $\delta \in [0, 1]$:

- Competitive factor

$$8\,\frac{1+\delta}{1-3\delta} + 1$$

- Guarantee to reach the door for

$$\delta \leq \frac{1}{3}$$

- Otherwise:
  drift may be greater than progress towards the door

Better Strategy?

- $f_i = 2^i$ is independent from δ
- Is there a strategy $F(\delta)$ that achieves a better competitive factor?
- YES!

Proof Outline

- Establish a competitive factor for a general strategy:
  - Split f_i into ℓ_i^+, ℓ_i^-: Covered distances to the right (left)
  - Assume a worst-case location for the door:
    Door is *slightly* missed in step $2j$ and hit in step $2j+2$.

  $\Rightarrow$ Worst-case values for ℓ_i^+ and ℓ_i^-:
  $\ell_i^- = (1+\delta)\, f_i$ and $\ell_i^+ = (1-\delta)\, f_i$

$\Rightarrow$ Functionals $G_{n,\delta}(F)$

- Gal, 1980: $f_i = \alpha^i$ minimizes $G_{n,\delta}(F)$

Worst-Case Aware Strategy

The strategy

$$f_i = \left(2\,\frac{1+\delta}{1-\delta}\right)^i$$

reaches the goal for *every* δ and achieves a factor of

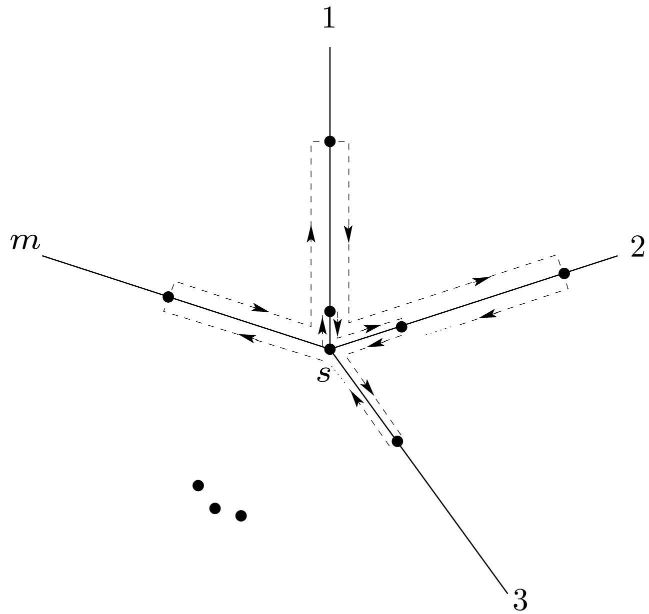
$$\frac{|\pi_{\mathsf{onl}}|}{d} \leq 1 + 8\,\left(\frac{1+\delta}{1-\delta}\right)^2\,,$$

which is optimal.

Extension to m rays

- Searching a goal on m rays
- No drift!
- $f_i = \left(\frac{m}{m-1}\right)^i$
- Independent from δ
- Optimal competitive factor:

$$3 + 2\,\frac{1+\delta}{1-\delta}\left(\frac{m^m}{(m-1)^{m-1}} - 1\right)$$



Summary

- Optimal strategies take the error into account.

- 2 Rays:

$$f_i = \left(2\,\frac{1+\delta}{1-\delta}\right)^i \qquad \text{yields} \qquad 1 + 8\left(\frac{1+\delta}{1-\delta}\right)^2$$

- m Rays:

$$f_i = \left(\frac{m}{m-1}\right)^i \qquad \text{yields} \qquad 3 + 2\,\frac{1+\delta}{1-\delta}\left(\frac{m^m}{(m-1)^{m-1}} - 1\right)$$

Thank you

for not searching the door during my talk ☺