The pledge algorithm reconsidered under errors in sensors and motion

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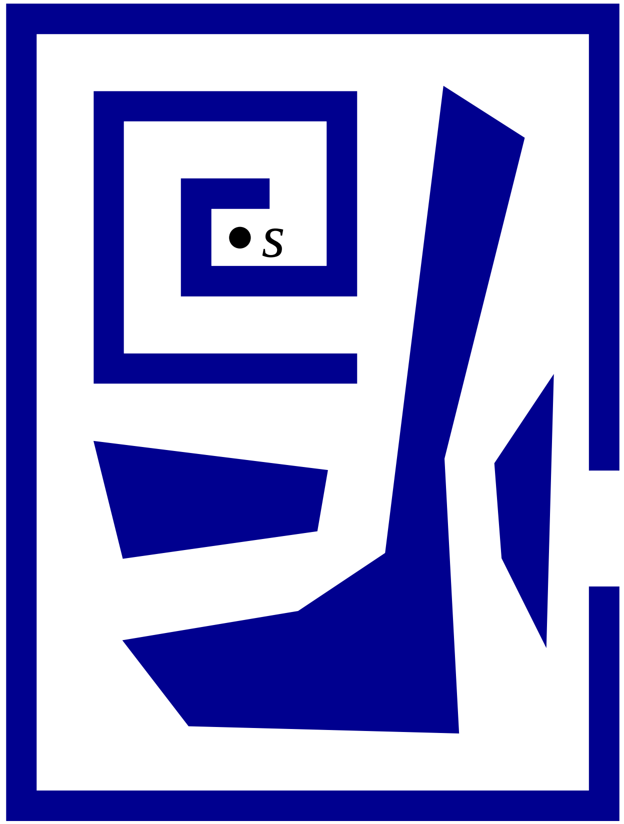
What is the Pledge algorithm?

Given

- A robot
  - Touch sensor
  - Angle counter
  - Move straight forward
  - Follow wall
- A maze (set of polygons)



Task

- Leave the maze

The Pledge algorithm

repeat

  $\omega = 0$

  repeat

    Move in direction ω in the free space

  until Robot hits an obstacle

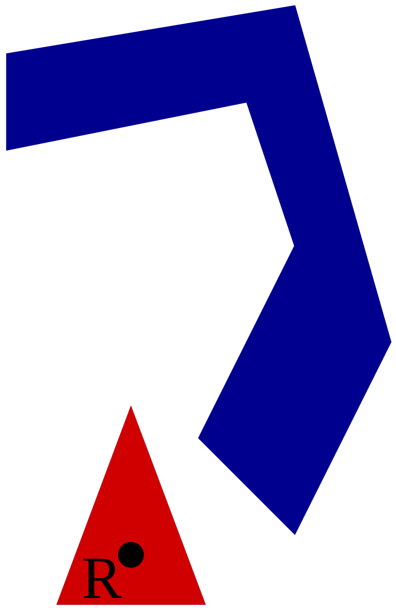
  repeat

    Follow the wall in counter-clockwise direction

    Count the overall turning angle in ω

  until Angle Counter $\omega = 0$

until Robot is outside the maze



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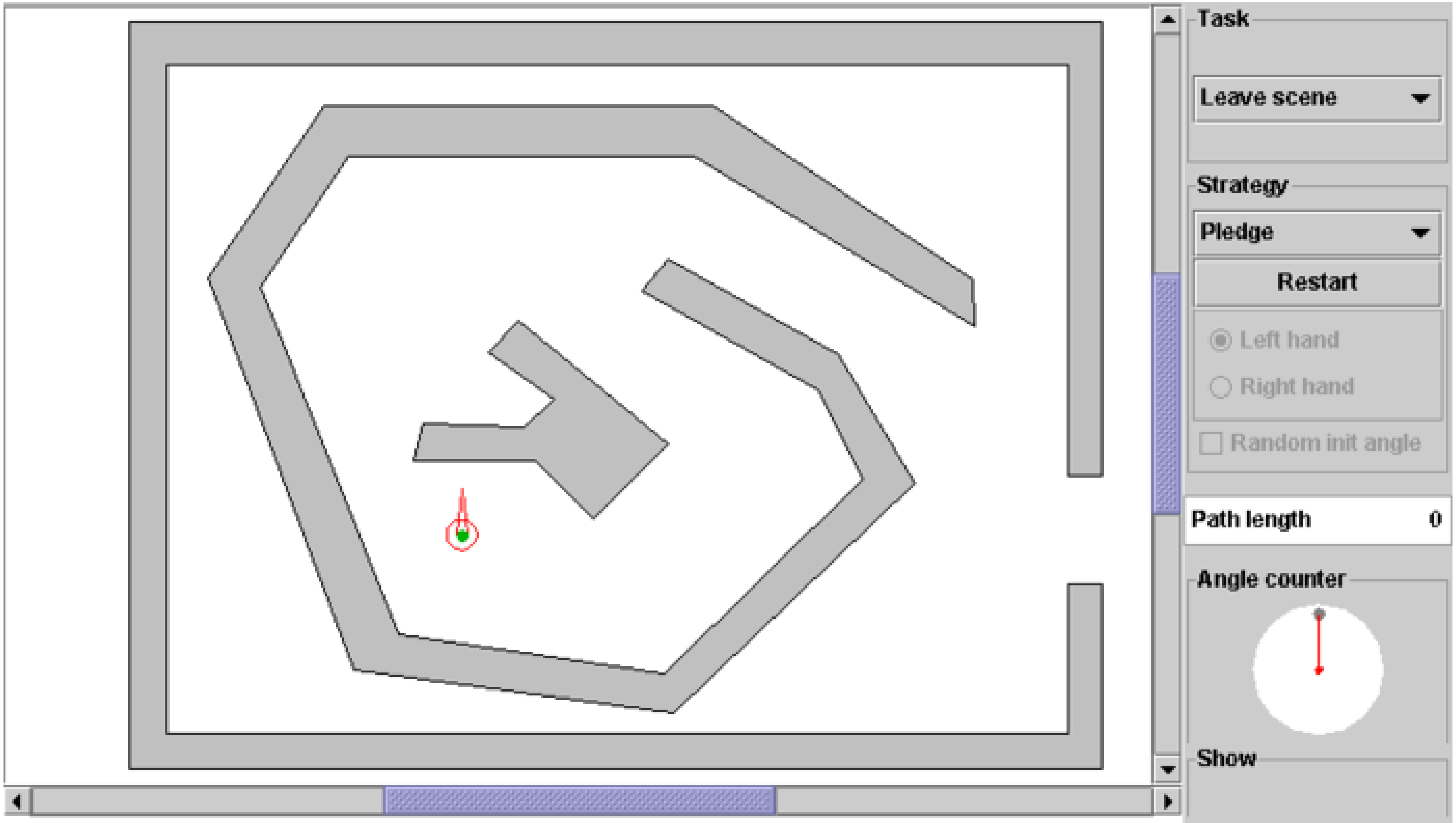

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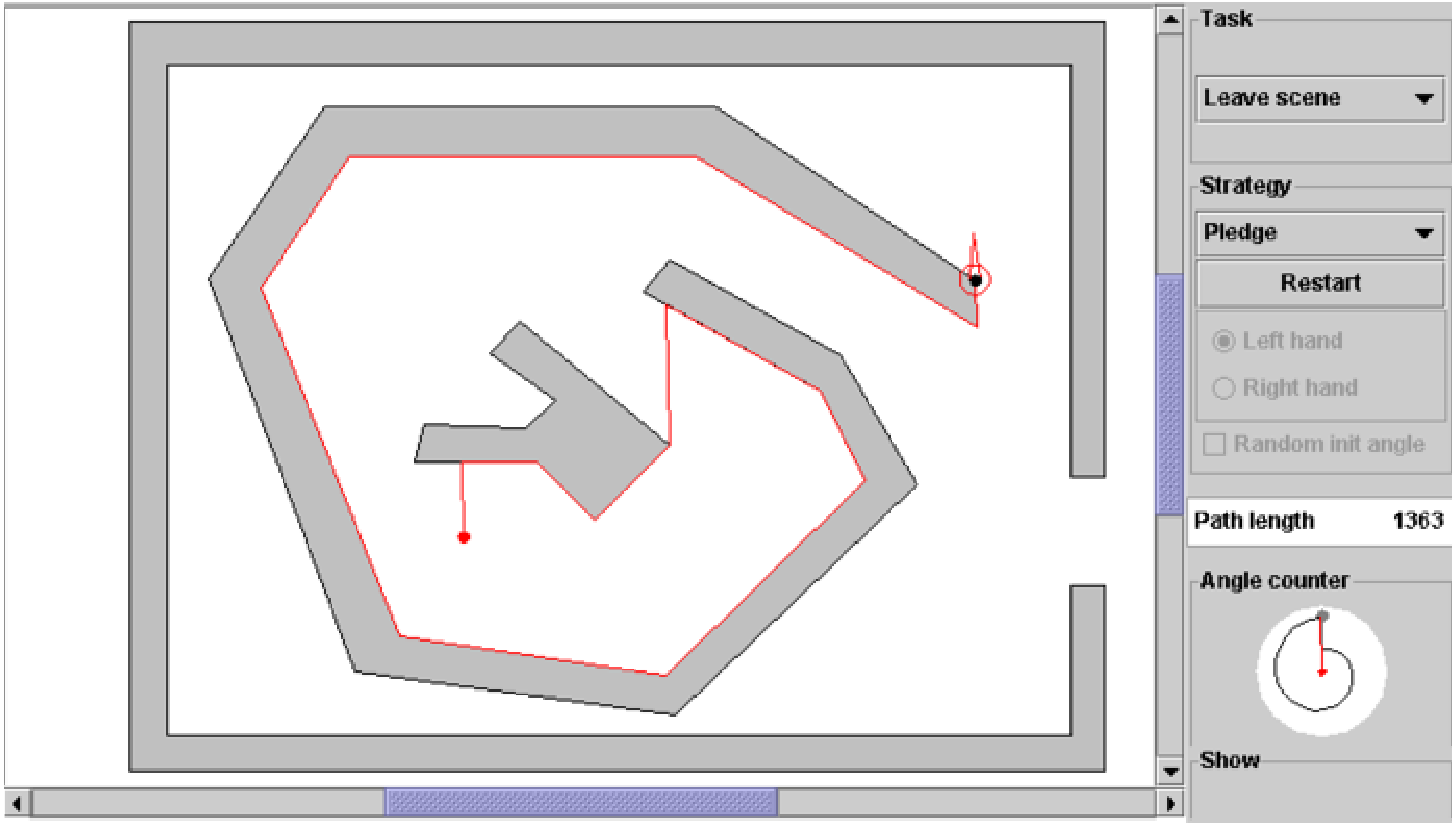

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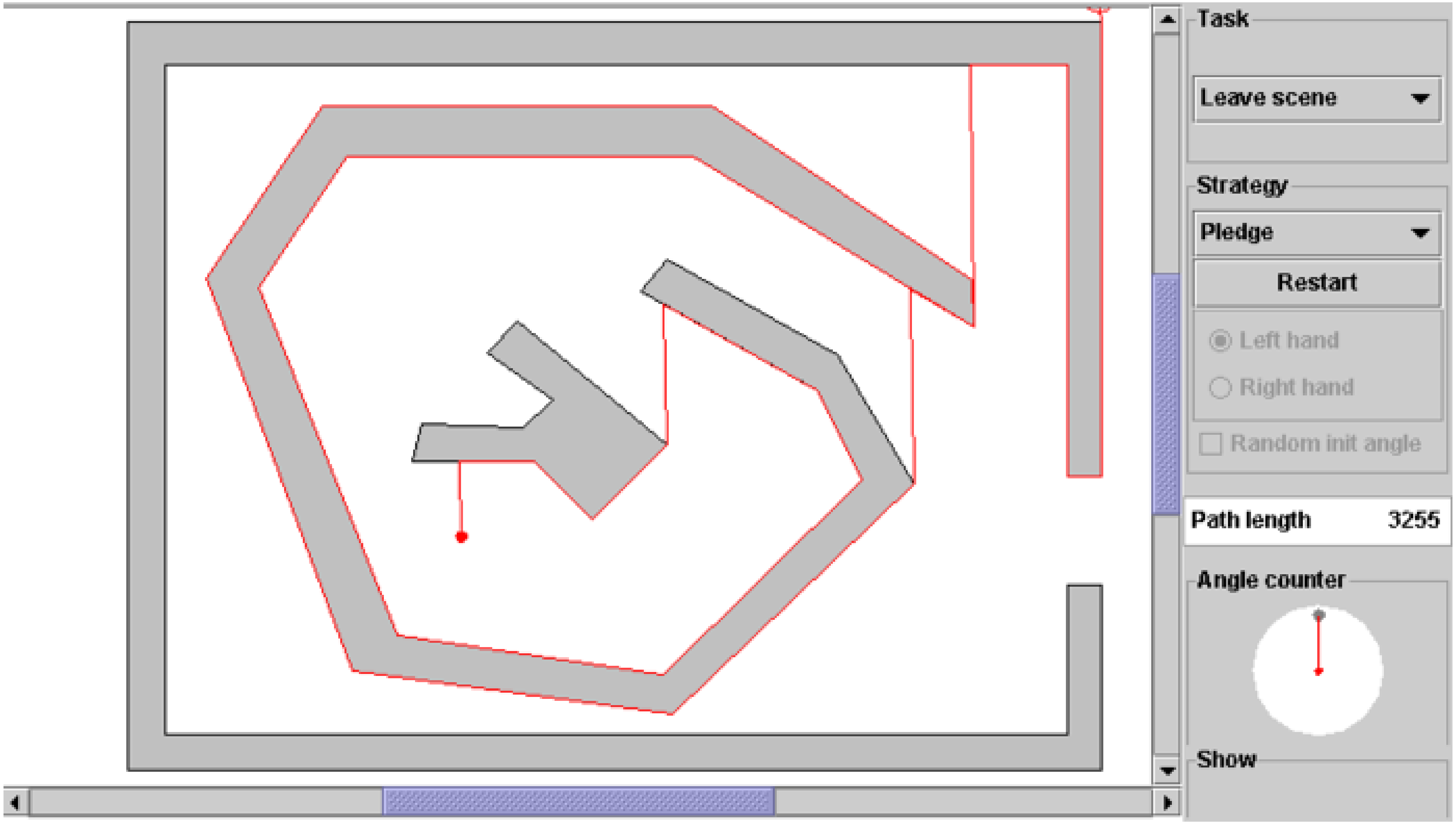


Why reconsidered?

Robotics:

$\oplus$ Real robots

$\oplus$ Consider errors in movement, sensors, computation

$\ominus$ Heuristic algorithms, statistical analysis

$\Rightarrow$ No guarantees

Computational geometry:

$\oplus$ Provable correctness, performance guarantees

$\ominus$ Idealistic assumptions (robot is error-free, robot is point-shaped)

$\Rightarrow$ Not implementable

Goal: Combine both approaches.

Correctness

Theorem 1. *(Abelson, diSessa, 1980)*
A robot will leave an unknown polygonal maze, provided that there is an exit.

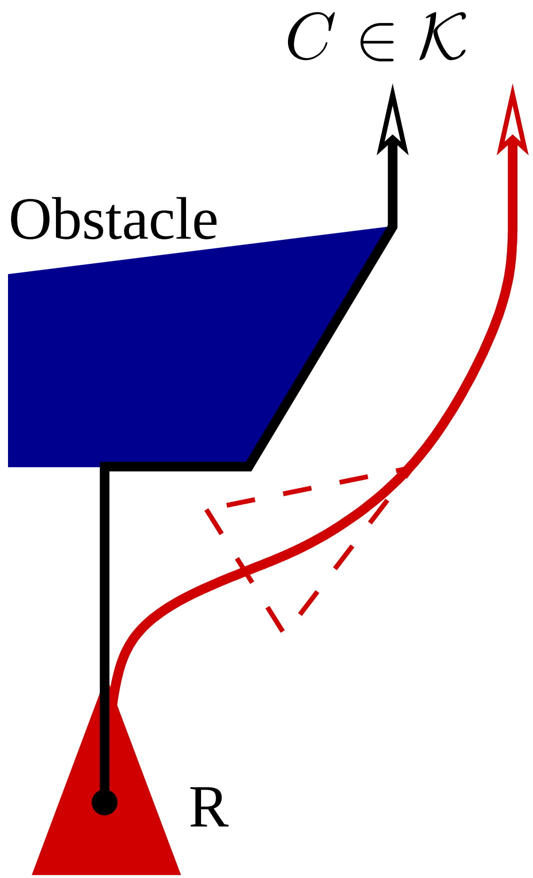
The proof relys on the assumptions:

- Robot is point-shaped
- Angle-counter is correct
- Straight motion is correct

But: What happens, if these assumptions cannot be fulfilled?

Idea

- Define a class $\mathcal{K}$ of curves in the robot's workspace
- $C \in \mathcal{K}$ represents possible path to an exit
- Robot will escape, if its strategy *follows* a $C \in \mathcal{K}$
- Find sufficient conditions for curves in $\mathcal{K}$

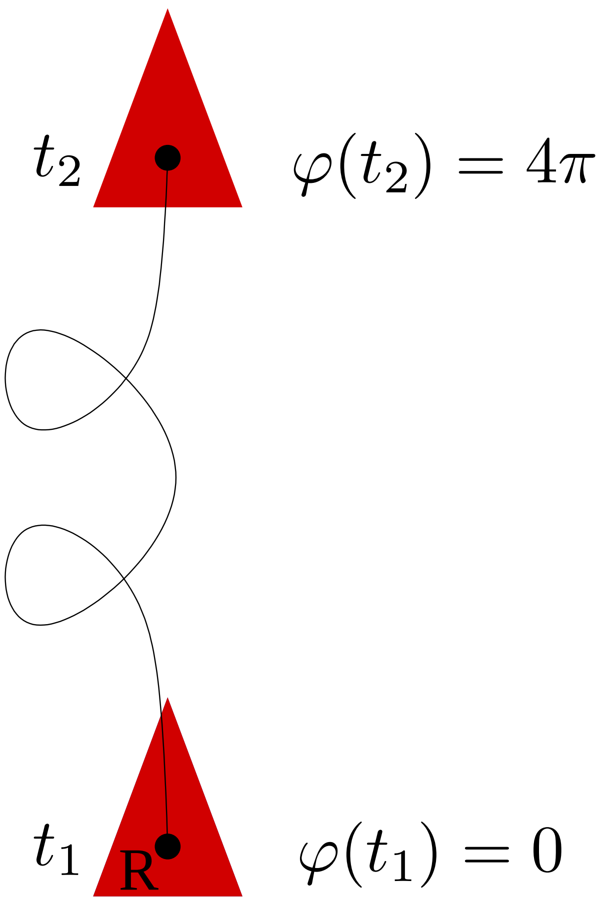


Preliminaries (1)

- Workspace $\mathcal{C} = \mathbb{R} \times \mathbb{R} \times \mathbb{R}$
- Curve $C(t) = (P(t), \varphi(t))$

  Position $P(t) = (X(t), Y(t))$

  Heading $\varphi(t) \in \mathbb{R}$ (!)



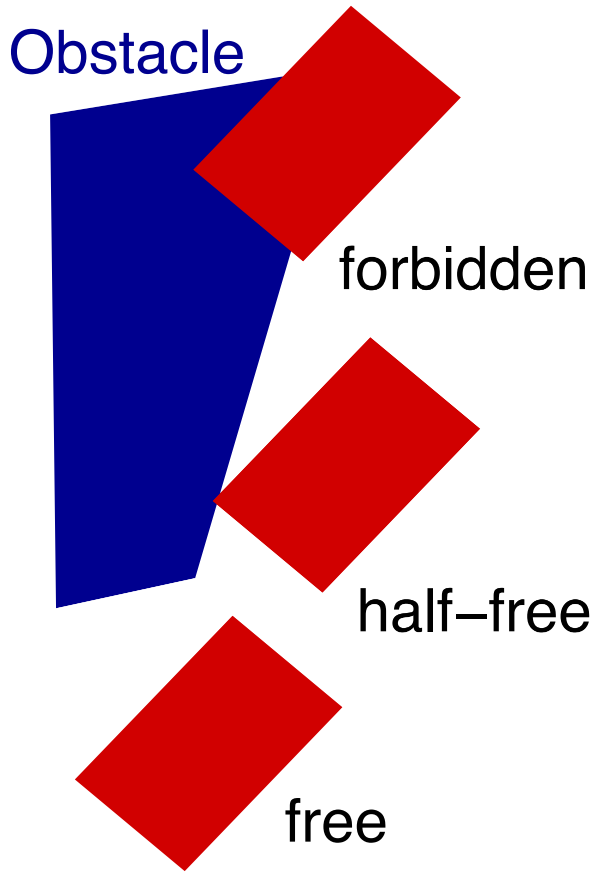
Preliminaries (2)

Points in the plane:

- Forbidden configuration ($\mathcal{C}_{\text{forb}}$):
  robot intersects obstacle

- Half-free configuration ($\mathcal{C}_{\text{half}}$):
  robot touches obstacle

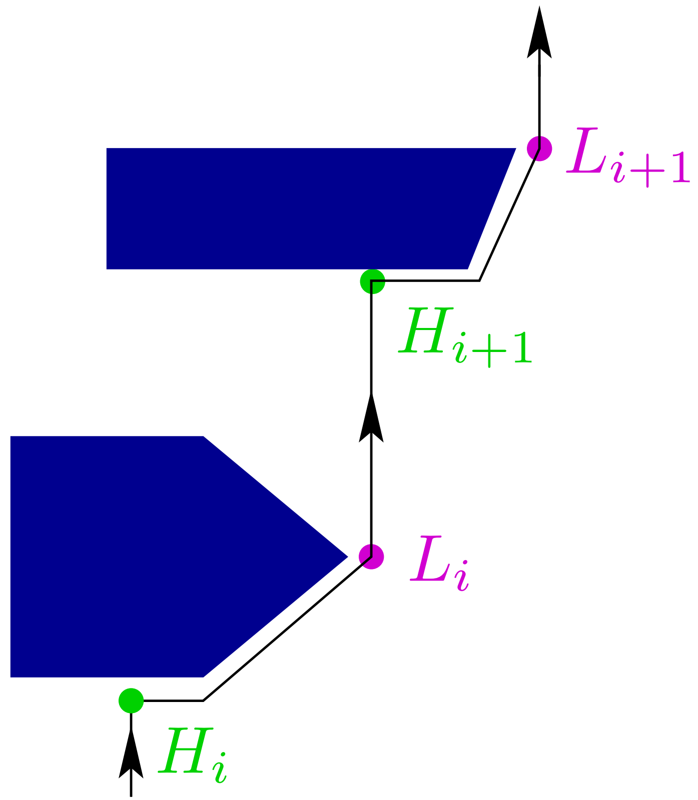
- Free configuration ($\mathcal{C}_{\text{free}}$):
  neither intersects nor touches



Preliminaries (3)

- Curve hits obstacle:
  Hit–Point H_i

- Curve leaves obstacle:
  Leave–Point L_i

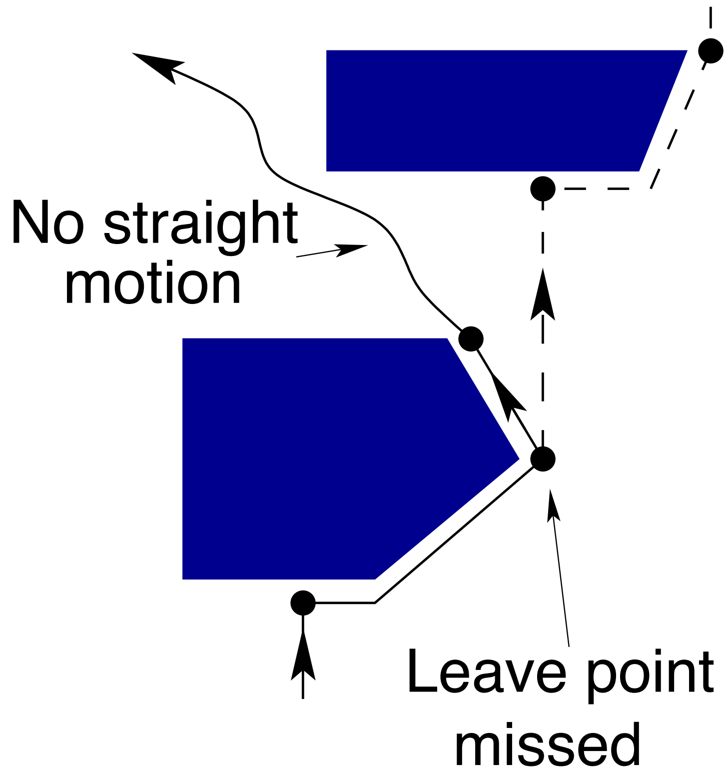


Types of errors

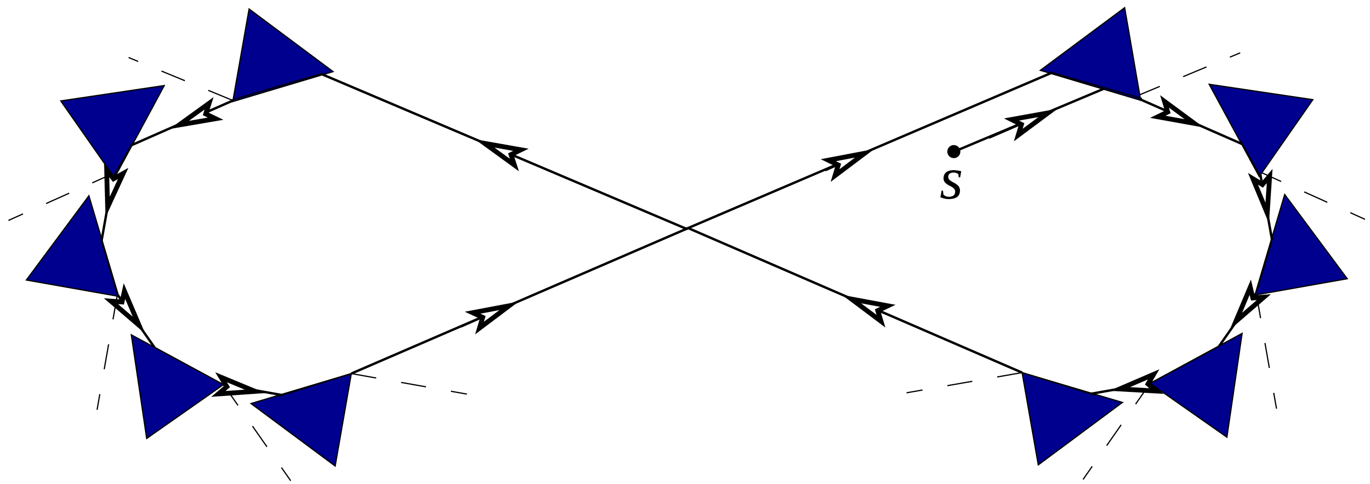
Types of movements in the pledge algorithm:

- straight motion through the free space
- following an obstacle wall while counting turning angles

Both types of movements may be afflicted with error.



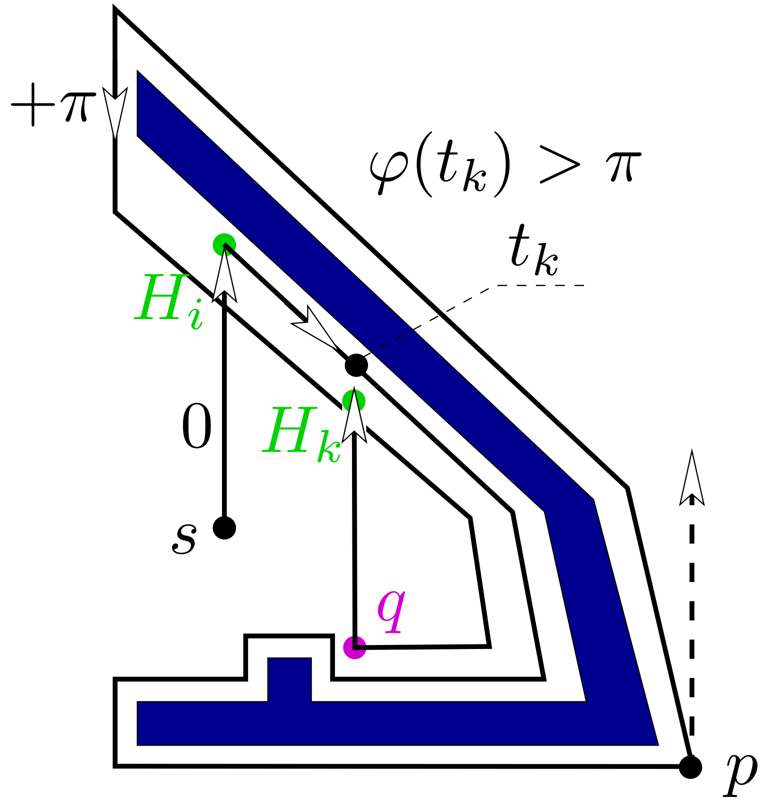
Free space condition



- Small deviations in the free space can sum up to big mistake
- Curve has to stay in a wedge around the initial direction
- $\forall t_1, t_2 \in C : P(t_1), P(t_2) \in \mathcal{C}_{\text{free}} \Rightarrow |\varphi(t_1) - \varphi(t_2)| < \pi$

Obstacle condition

- Leave point is missed
- $\Rightarrow$ angle counter 'overwinded'
- $\forall H_i, t \in C$:
  $P(t) = P(H_i)$
  $\Rightarrow \varphi(t) - \varphi(H_i) < \pi$



Sufficient conditions

Definition 2. *Let $\mathcal{K}$ be the class of curves in $\mathcal{C}_{free} \cup \mathcal{C}_{half}$ that satisfy the following conditions:*

(i) The curve circles an obstacle in a counter-clockwise direction.

(ii) Every leave point belongs to a vertex of an obstacle.

(iii) Free space condition:

$$\forall t_1, t_2 \in C : P(t_1), P(t_2) \in \mathcal{C}_{free} \Rightarrow |\varphi(t_1) - \varphi(t_2)| < \pi$$

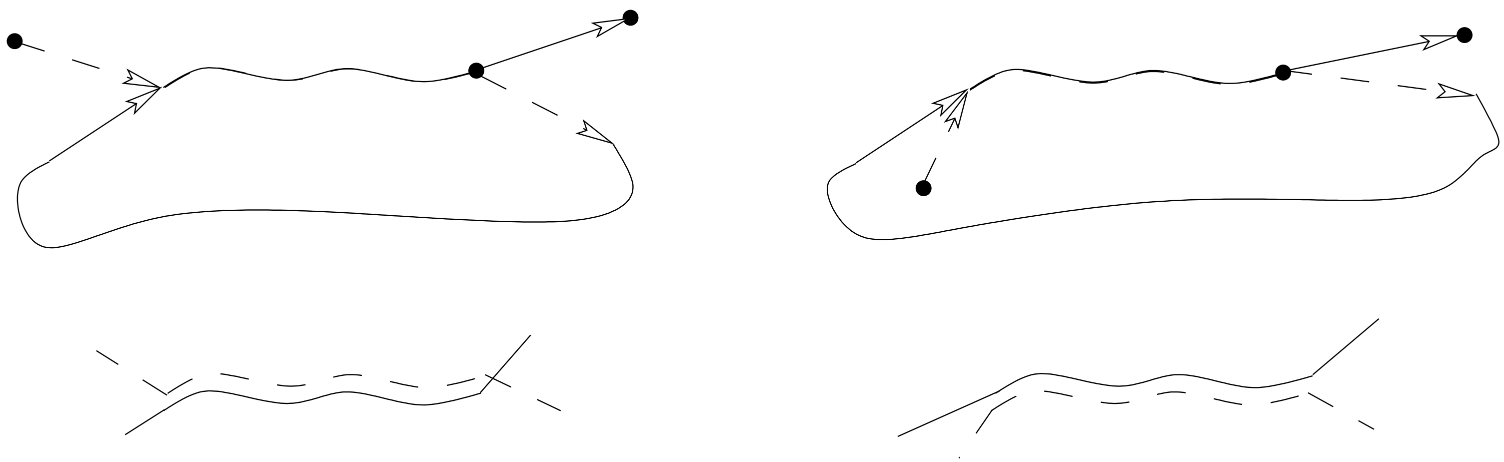
(iv) Obstacle condition:

$$\forall H_i, t \in C : P(t) = P(H_i) \Rightarrow \varphi(t) - \varphi(H_i) < \pi$$

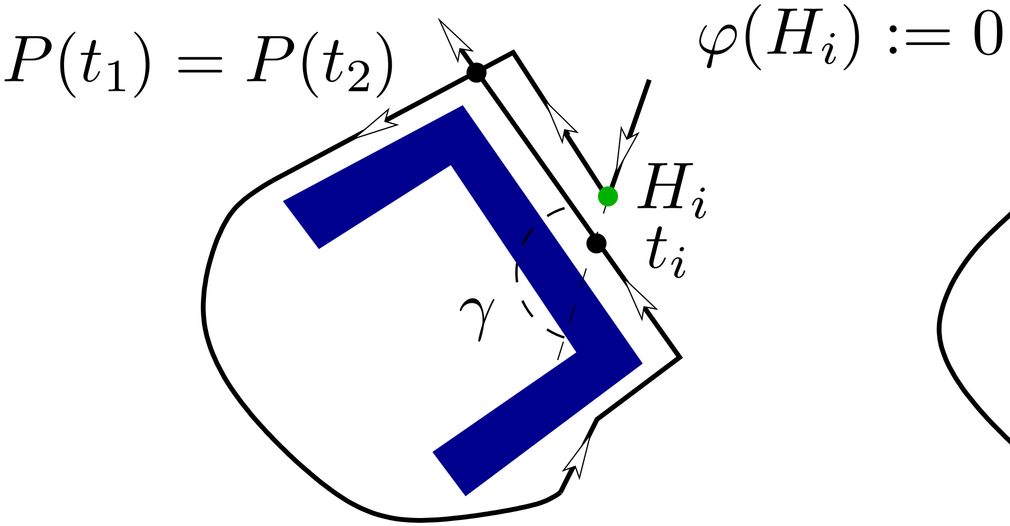
No crossings

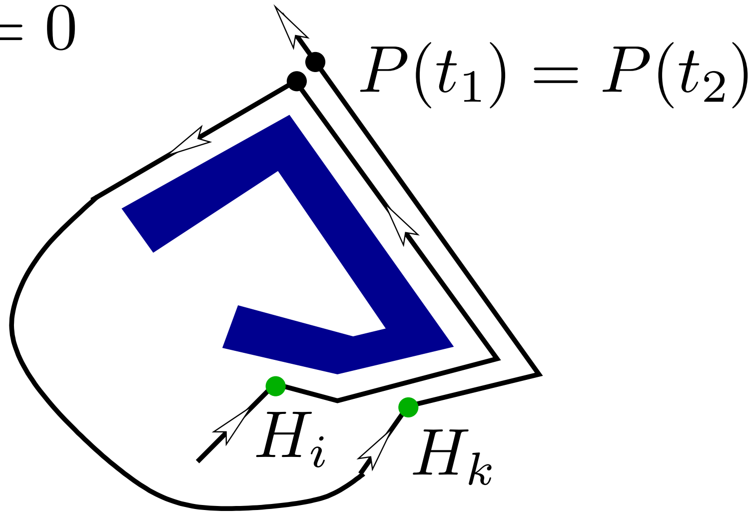
Lemma 3. *A curve $C \in \mathcal{K}$ cannot cross itself.*

crossing **no crossing, just touching**



Proof, counterclockwise loop





$P(H_i)$ is met again at $t_i > H_i$

1) Full turn $\Rightarrow \varphi(t_i) - \gamma = 2\pi$

2) Turning angle $-\pi < \gamma < 0$

$\Rightarrow \varphi(t_i) - \varphi(H_i) > \pi$ ↯

$P(H_i)$ is not met again

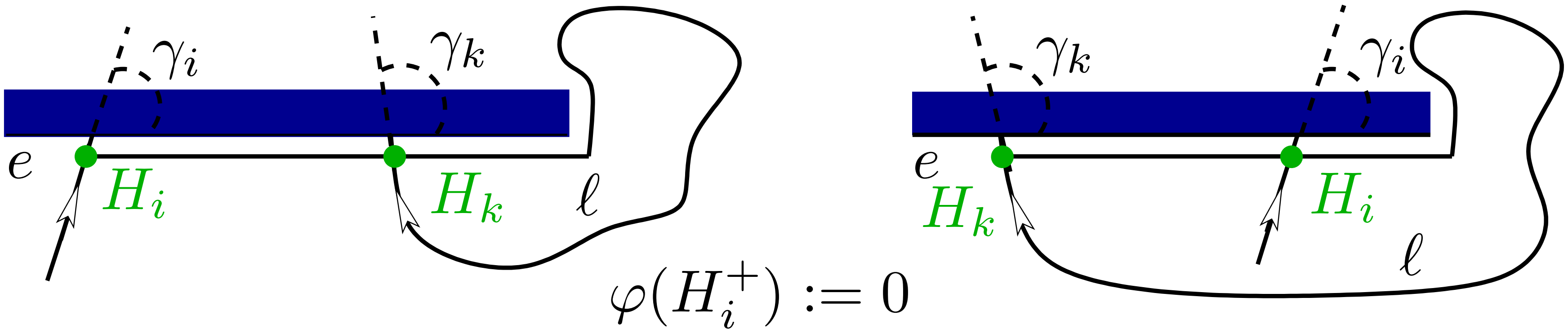
$\Rightarrow$ no crossing

Only one hit per edge

Lemma 4. *A curve* $C \in \mathcal{K}$ *will hit every edge in the environment at most once.*

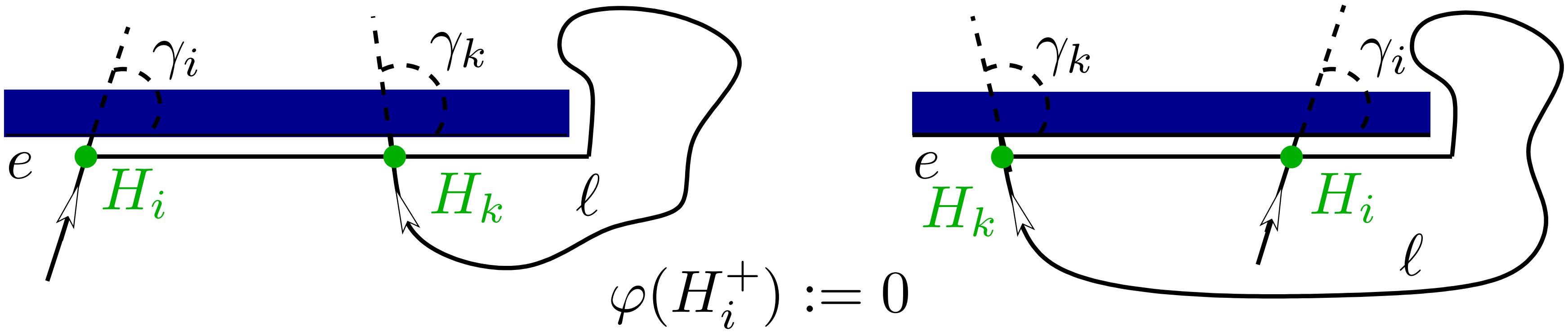
Proof: Assume C hits e twice.

Proof (1)



- Heading after the robot turns: $\varphi(H_{i/k}^+) := \varphi(H_{i/k}) + \gamma_{i/k}$
- Turning angles: $-\pi < \gamma_i, \gamma_k < 0$
- Curve follows $e \Rightarrow \varphi(H_k^+) = 2k\pi, k \in \mathbb{Z}$
- $k \neq 0 \Rightarrow |\varphi(H_k) - \varphi(H_i)| = |2k\pi - \gamma_k + \gamma_i| > \pi$ ↯

Proof (2)



- $k = 0 \Rightarrow \varphi(H_k^+) = 0$
- Loop has no crossings

  $\Rightarrow \pm 2\pi$ turn in loop ℓ

  $\Rightarrow \varphi(H_k^+) = \pm 2\pi$ ↯

Main theorem

Theorem 5. *A robot, whose path follows a curve $C \in \mathcal{K}$, will escape from an unknown maze, if this is possible at all.*

Proof.

- Curve hits every edge at most once
- After the curve has visited every edge, the robot must escape or there is no exit

$\square$

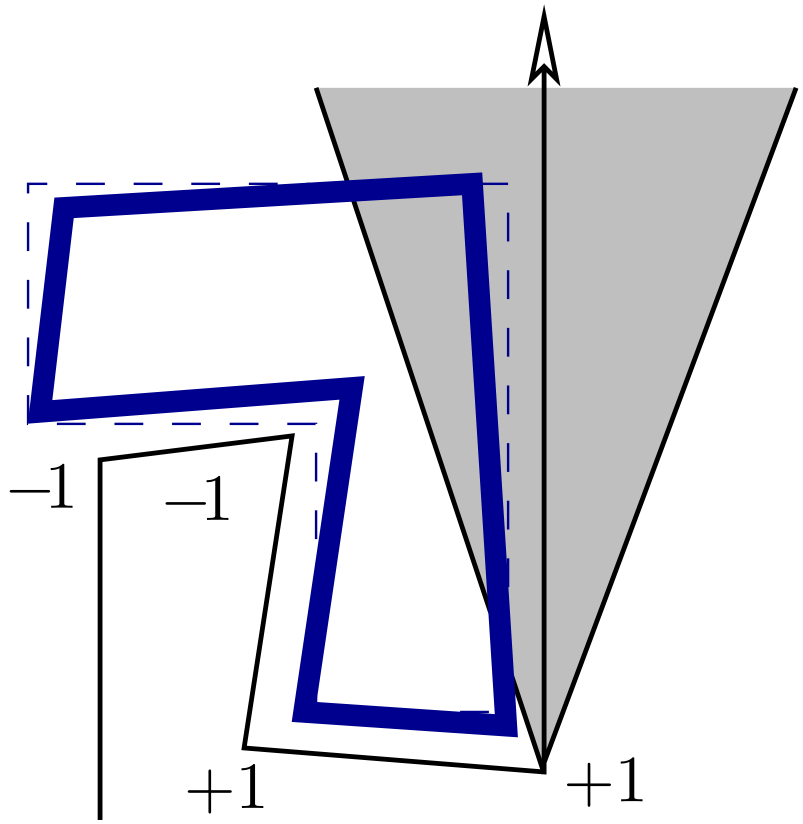
Conclusion (1)

- Robot is able to move straight forward exactly
- $\beta :=$ maximal difference between real angle and measured angle
- $n :=$ Number of edges in the environment
- Robot escapes, if

$$|\beta| < \frac{\pi}{n^2}$$

Conclusion (2)

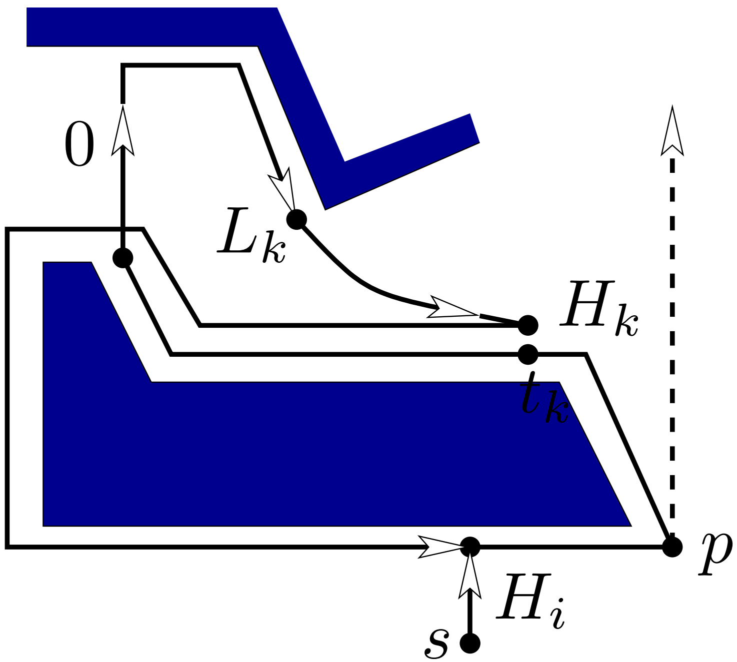
- "Almost rectangular" environment
- Strategy: just count convex/concave vertices
- Robot escapes, if it guarantees its heading in the free space up to an angle α
- α depends on the deviations from the exact rectangular environment



Thank you!

Obstacle Condition (2)

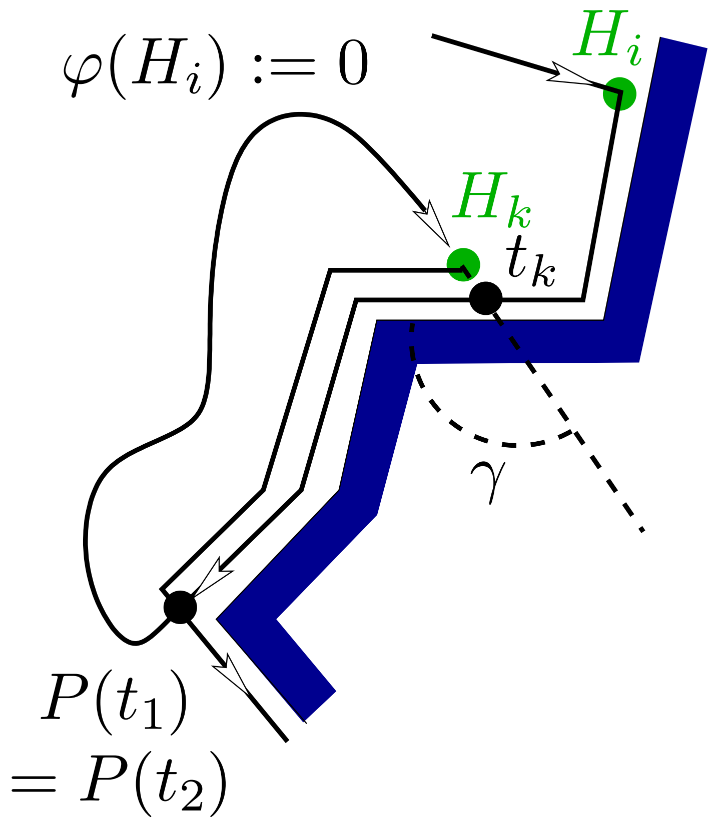
- Leave point is missed
- $\varphi(t_k) = \frac{\pi}{2}$
- $\varphi(H_k) < -\frac{\pi}{2}$
- Both errors sum up to $> \pi$
- $\forall H_i, t \in C :$
  $P(t) = P(H_i)$
  $\Rightarrow \varphi(t) - \varphi(H_i) < \pi$



$\varphi(t_k) = \frac{\pi}{2}$
$\varphi(H_k) < -\frac{\pi}{2}$

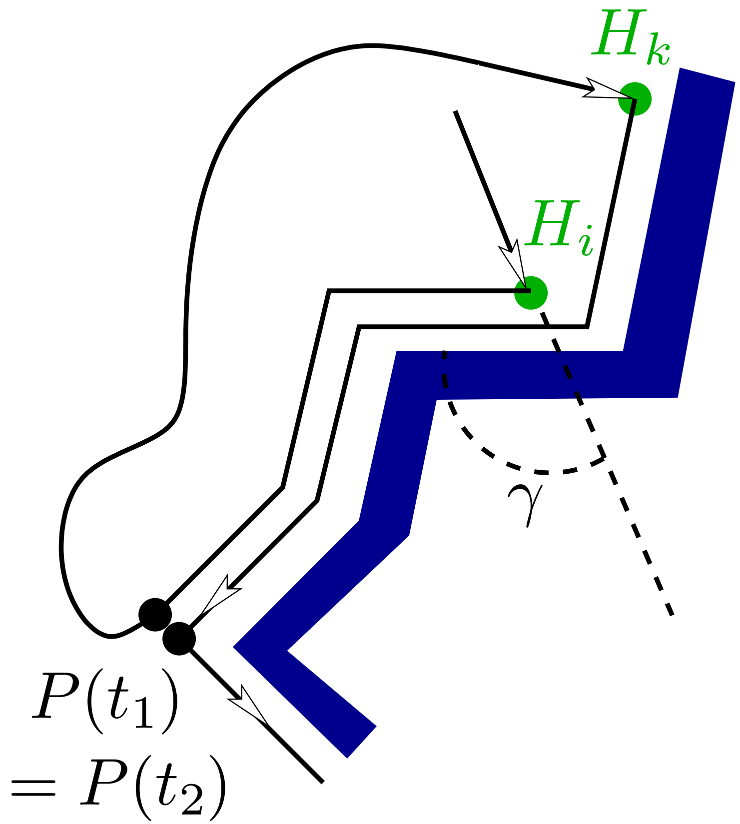
Proof, clockwise loop (1)

- $\varphi(H_k^+) = \varphi(H_k) + \gamma$
- $-\pi < \gamma < 0$
- $P(H_k)$ was already met
  $\Rightarrow$ Full turn: $\varphi(H_k^+) = \varphi(t_k) - 2\pi$
- Obstacle Condition:
  $\varphi(t_k) - \varphi(H_k) < \pi$
  $\Leftrightarrow \varphi(H_k^+) + 2\pi - \varphi(H_k) < \pi$
  $\Leftrightarrow \varphi(H_k) + \gamma + 2\pi - \varphi(H_k) < \pi$
  $\Leftrightarrow \gamma < -\pi$ ↯

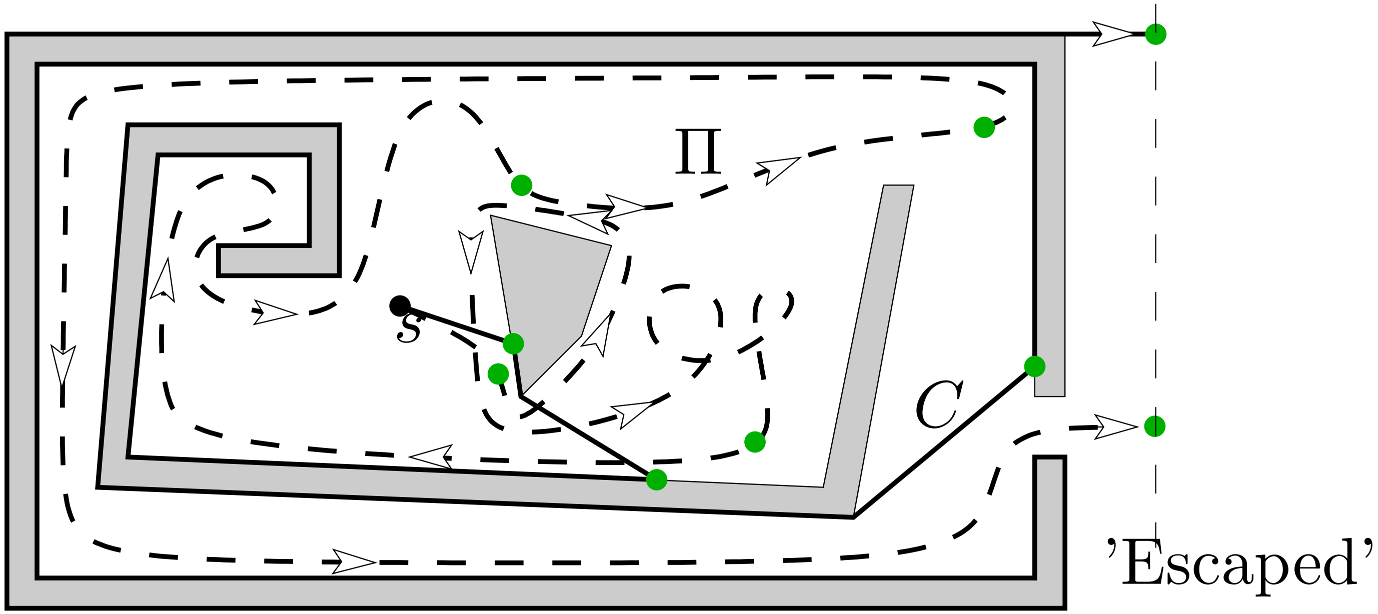


Proof, clockwise loop (2)

- $P(H_k)$ was not met before
- $\Rightarrow$ no crossing



Following a curve



Corollary 6. *A robot escapes, if* $\exists C \in \mathcal{K}$, *such that the sequence of hit points of* C *is a subsequence of the hit points generated by the robot's strategy.*