

## Variables

**Slot assignment variables** ( $x_{si}$ ) module  $m_i$  is scheduled in slot  $s$

**Time assignment variables** ( $y_{tij}$ ) request  $(i, j)$  is scheduled at time  $t$

**Occupancy variables** ( $z_{stij}$ ) keep track of occupied slots

**Usage variables** ( $u_t$ ) indicates which times steps are used

## Constraints

$$\text{Assignment Constraints } \sum_{s=1}^N x_{si} = 1 \quad \forall i = 1, \dots, M, \quad (1)$$

$$\sum_{t=1}^T y_{tij} = 1 \quad \forall i = 1, \dots, M, j = 1, \dots, \ell_i. \quad (2)$$

$$\text{Boundary Constraints } \forall i, j, s = s_{\text{low}}, \dots, s_{\text{up}} : x_{si} = 0 \quad (3)$$

$$\text{Order Constraints } \sum_{t=1}^T t y_{tij} - \sum_{t=1}^T t y_{tij-1} > 0 \quad \forall i, j > 0. \quad (4)$$

$$\text{Occupancy Constraints } \forall i = 1, \dots, M, j = 1, \dots, \ell_i, s = 1, \dots, N, t = 1, \dots, T, \\ s' = s_{\text{low}}, \dots, s_{\text{up}} : x_{si} + y_{tij} - z_{s'tij} \leq 1 \quad (5)$$

$$\text{Exclusive Constraints } \forall t = 1, \dots, T, s = 1, \dots, N : \sum_{i=1}^M \sum_{j=1}^{\ell_i} z_{stij} \leq 1 \quad (6)$$

$$\text{Delay Constraints } \forall i = 1, \dots, M, j = 1, \dots, \ell_i - 1, s = 1, \dots, N, \\ t = 1, \dots, T - 1 : z_{stij} - z_{s(t+1)ij} - y_{(t+1)i(j+1)} \leq 0 \quad (7)$$

$$\text{Usage Constraints } \forall t = 1, \dots, T, i = 1, \dots, M, j = 1, \dots, \ell_i : u_t - y_{tij} \geq 0 \quad (8)$$

$$\forall t = 2, \dots, T : u_{t-1} - u_t \geq 0. \quad (9)$$

## Objective Function

$$\min \sum_{i=1}^T i u_i$$

subject to

Eq. (1)–(9)

$$x_{si} \in \{0, 1\}, y_{tij} \in \{0, 1\}, z_{stij} \in \{0, 1\}, u_t \in \{0, 1\}$$