Exploring Simple Grid Polygons

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29th August 2005

- Robot, R, has to explore an unknown environment, P
- More precisely, find a tour that
  - visits every part of P at least once
  - returns to the robot's start point
  - can be computed online
  - is as short as possible
- For example: lawn mowing, cleaning

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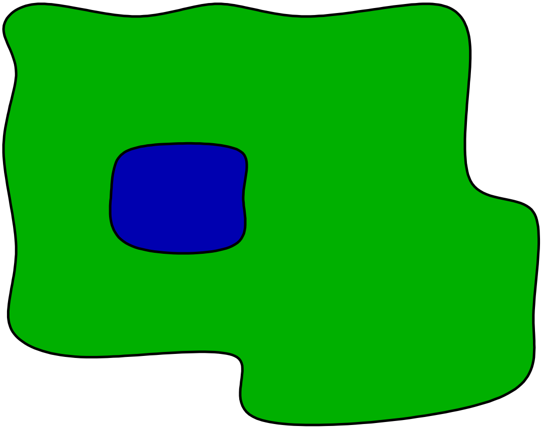
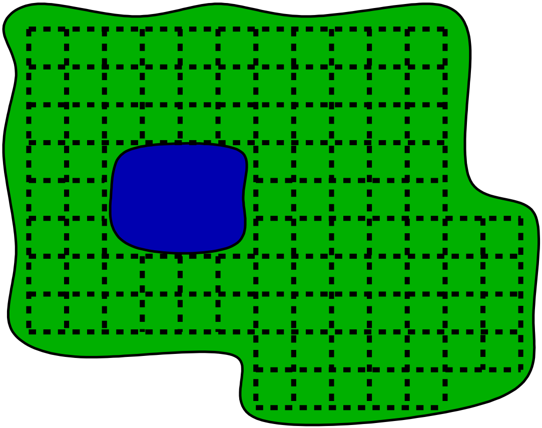


Grid polygon:

- Environment is subdivided by an integer grid
- Simple $\Rightarrow$ No holes

Robot

- No vision
- Can sense 4 adjacent cells
- Can enter adjacent, *free* cell



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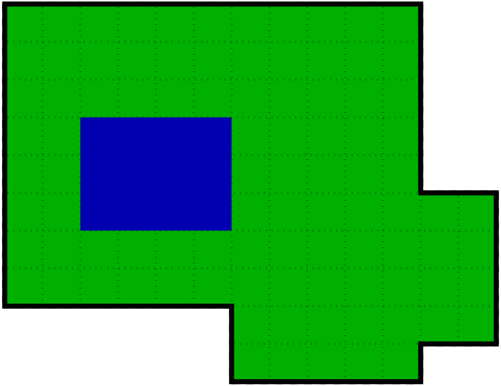

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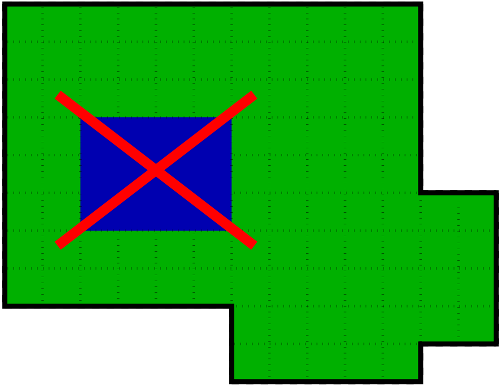

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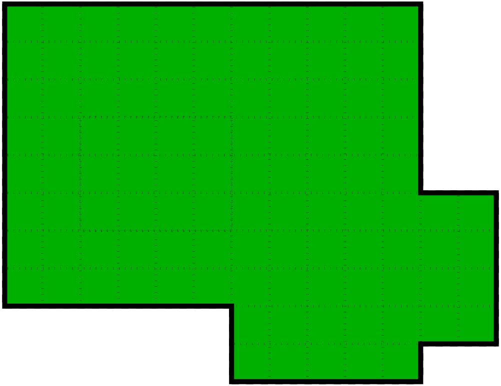

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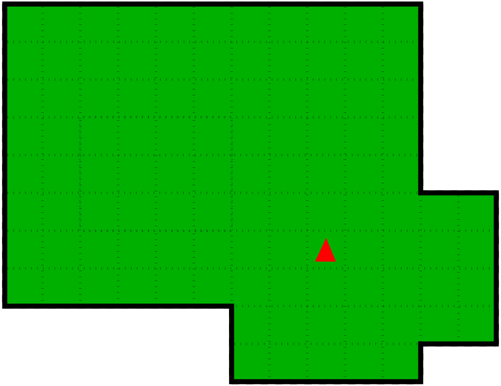

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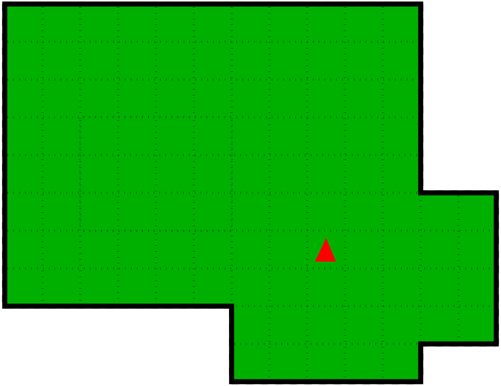

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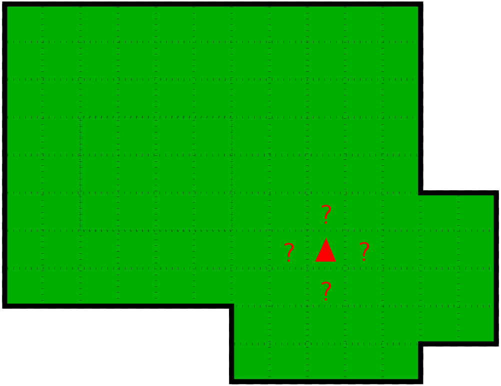

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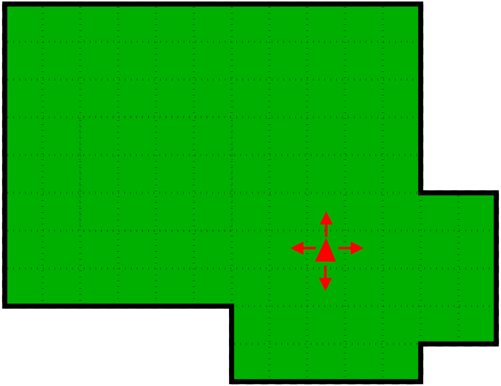

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Offline (i. e., environment is known to the robot)

- With holes:
  NP-hard [Itai, Papadimitriou, Szwarcfiter; 1982]
- Without holes:
  $\frac{4}{3}$-approximation [Ntafos; 1992]
  $\frac{6}{5}$-approximation [Arkin, Fekete, Mitchell; 2000]

Online

- With holes:
  [Icking, Kamphans, Klein, Langetepe; 2000]
  [Gabriely, Rimon; 2000]

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Why Simple Polygons?

Theorem (IKKL; 2000)

Lower bound on the online exploration of grid polygons with *holes: 2.*

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There is a $\frac{4}{3}$-competitive online exploration strategy for polygons without *holes.*

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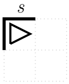
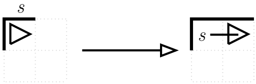
Theorem

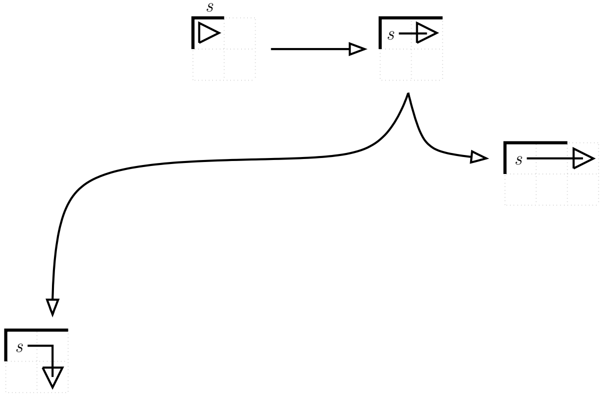
No online exploration strategy achieves a factor better than $\frac{7}{6}$ in simple grid polygon.


s

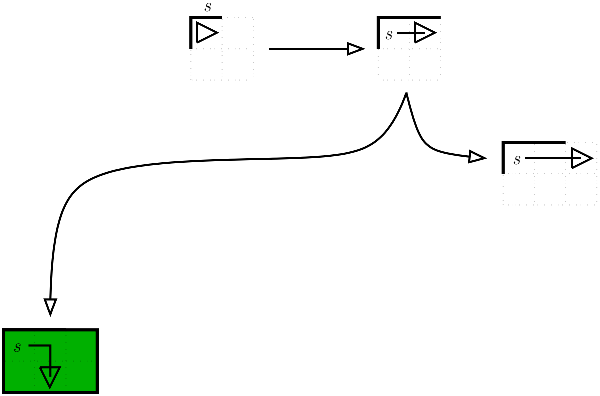
w. l. o. g.: East



South or East

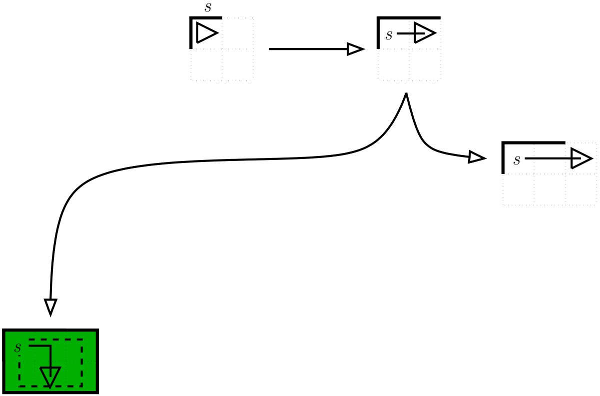


Close Polygon



Proof: Lower Bound

Online vs. Optimal



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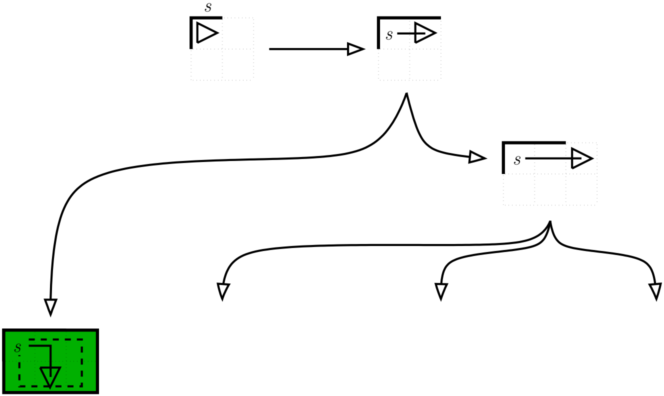
3 Possibilities:



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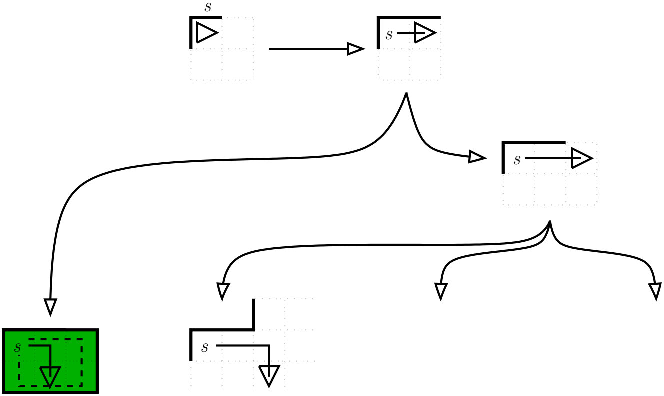
3 Possibilities: South,



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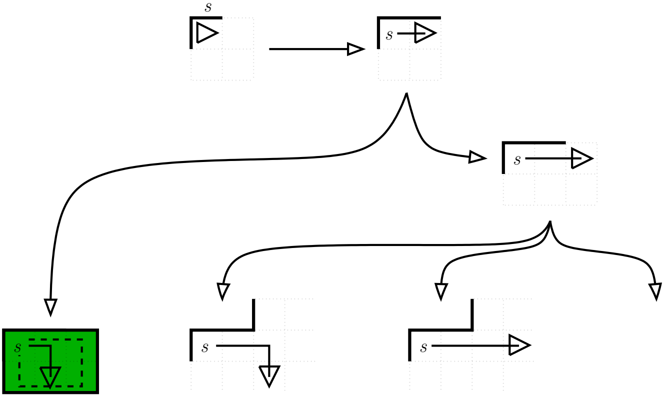

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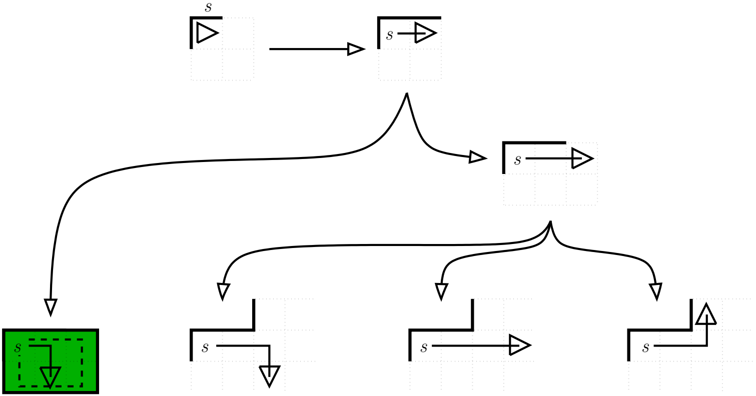
3 Possibilities: South, East, North



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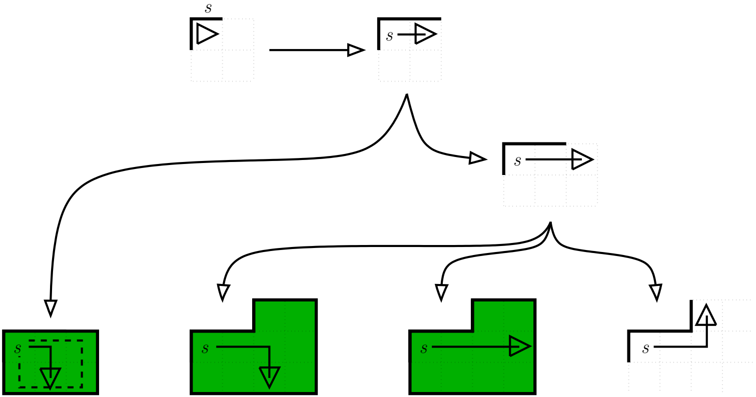

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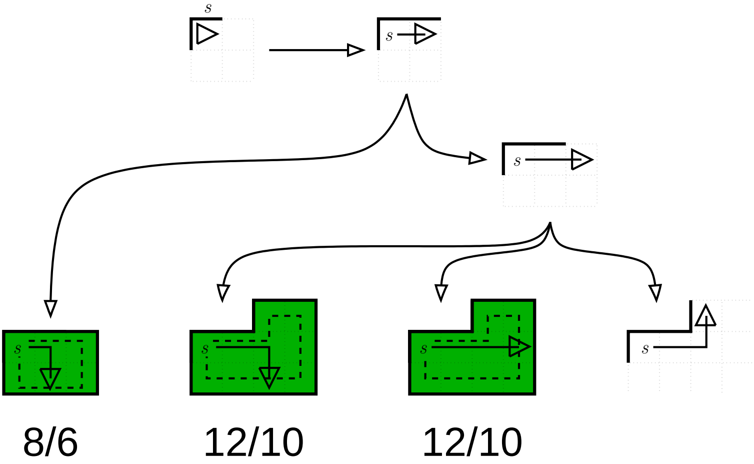

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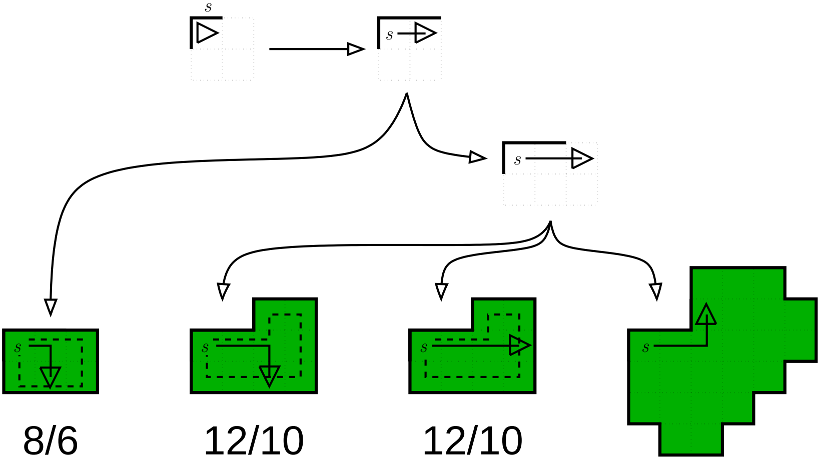

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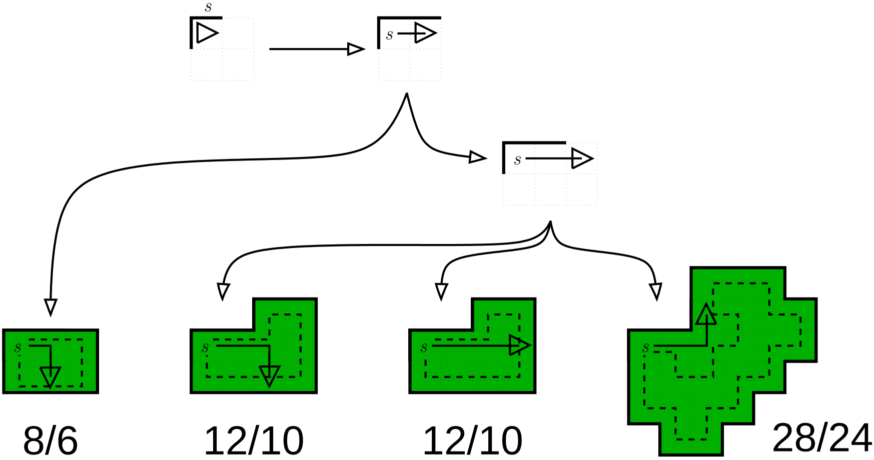
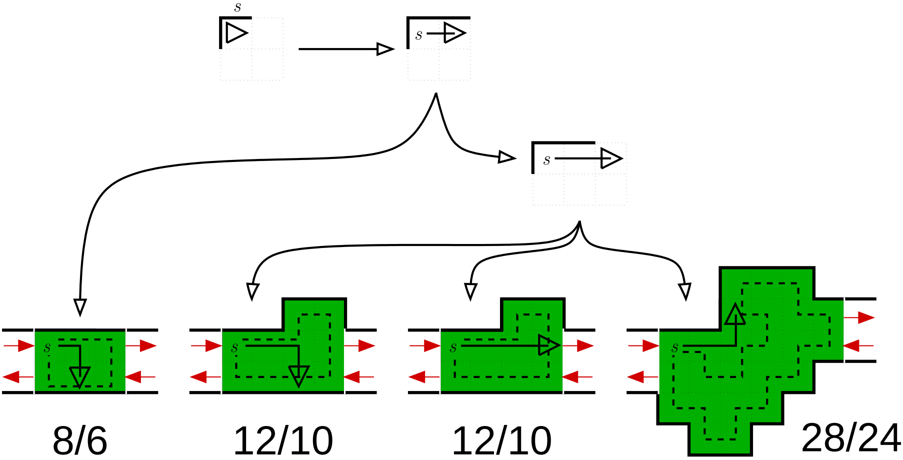
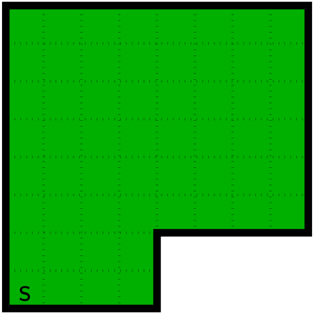


Polygons of arbitrary size



SmartDFS: An exploration strategy (1)



- First idea: Apply depth-first search (DFS)
- *Left-hand rule*: prefer step to the left over a straight step over a step to the right
- Visits *each* cell twice!

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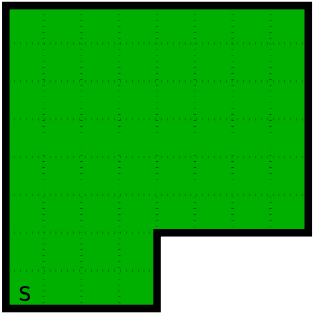

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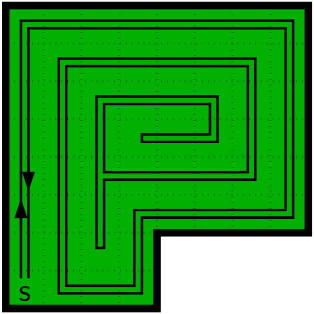

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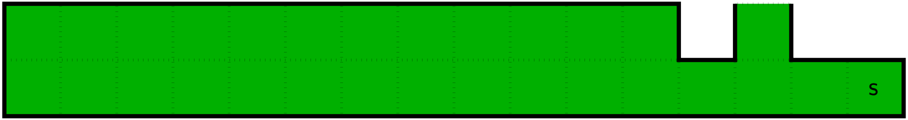

- DFS visits each cell twice
- More reasonable: Return directly to unvisited cell
- Improved DFS

Improvement 1

Return directly to those cells that have unexplored neighbors.

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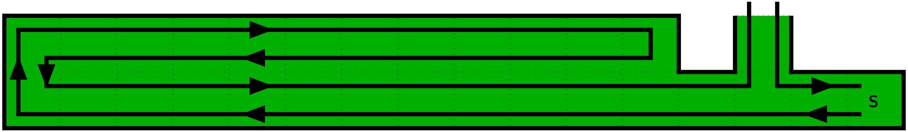

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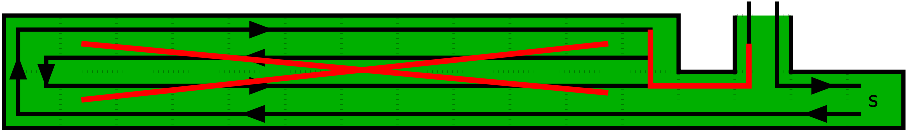

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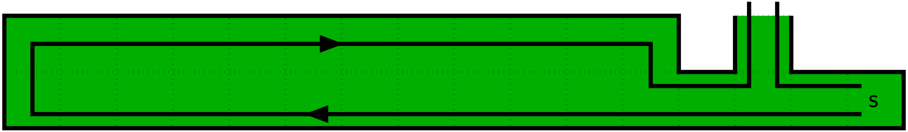

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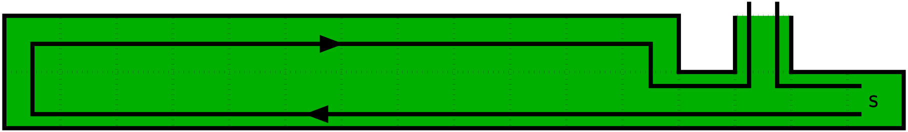

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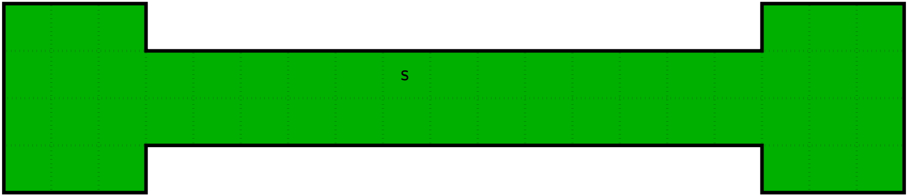

- DFS visits long corridor four times
- More reasonable: Visit right part immediately, continue with the corridor, visit left part, return to s
- Long corridor is traversed only two times!
- *Split cells*: Set of unvisited cells gets disconnected

Improvement 2

Detect and handle split cells (i. e., prefer parts of P farther away from the start).

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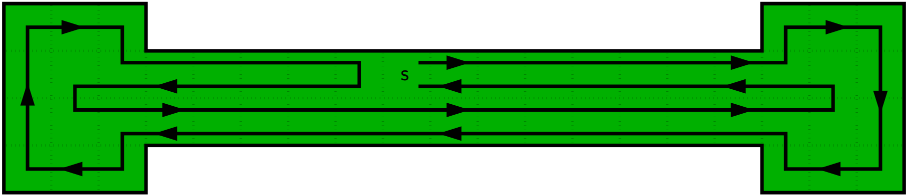

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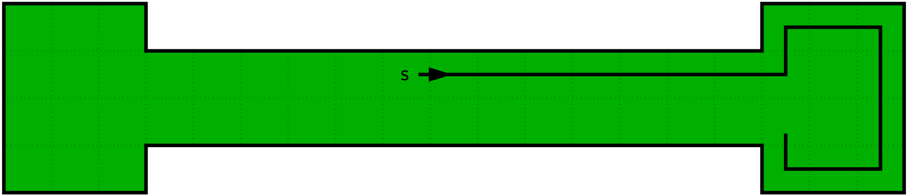

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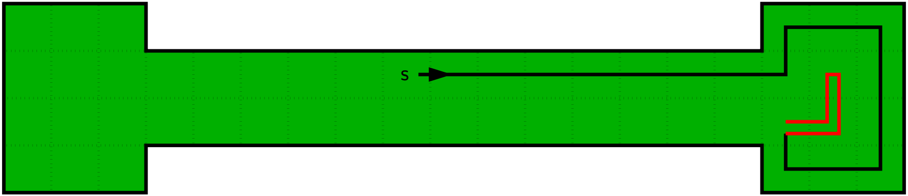

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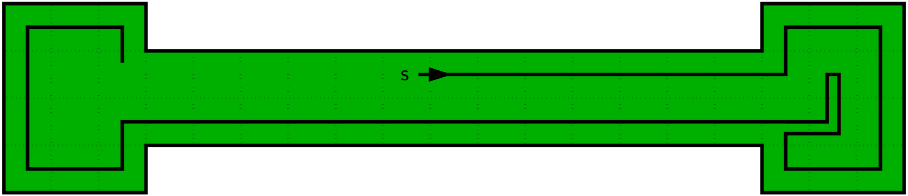

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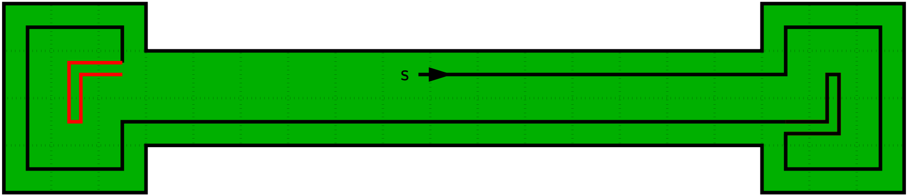

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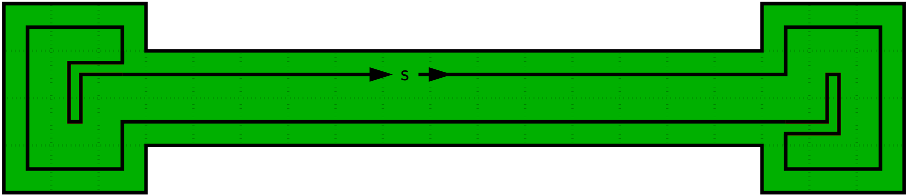

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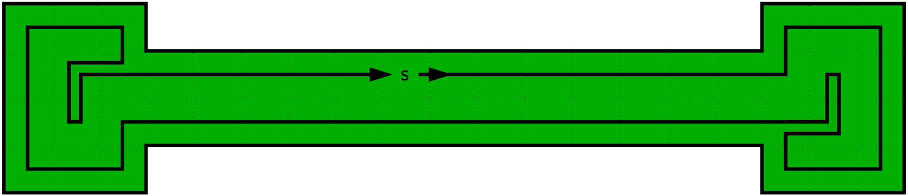

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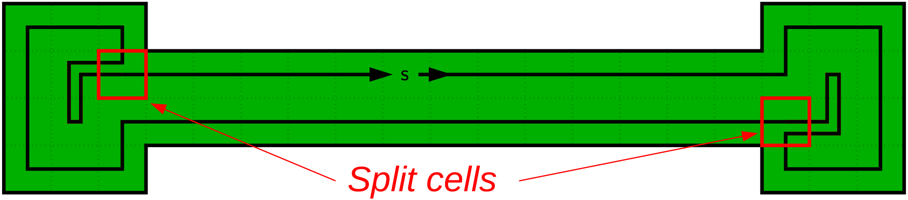

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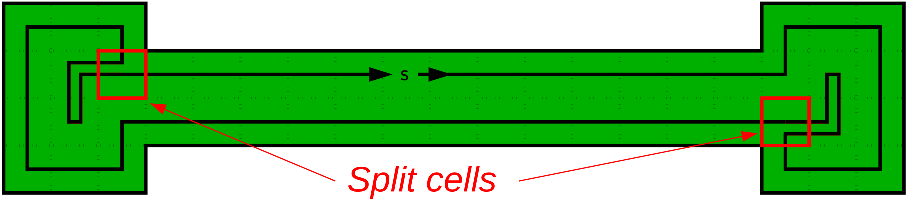

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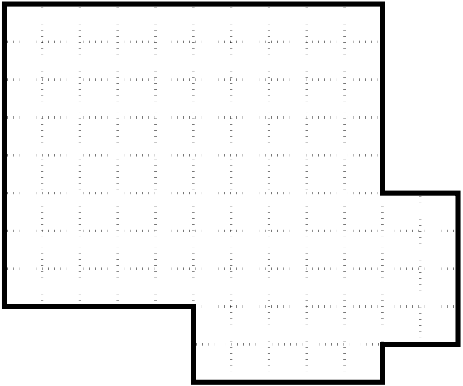
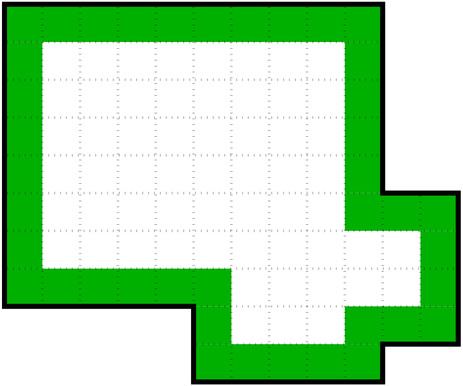
`http://www.geometrylab.de/Gridrobot/`



- *First layer* :=
  Boundary cells of P
- *1-offset* :=
  P without first layer
- Analogously: *Second layer*
- *2-offset*
- and so on
- E: #edges between free and blocked cells

Lemma (Number of edges)

P' *is* ℓ*-offset of* $P \Rightarrow E(P') \leq E(P) - 8\ell$.



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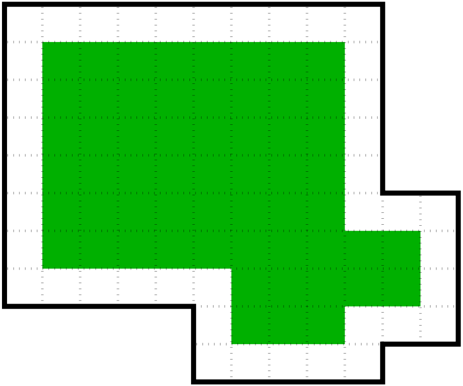

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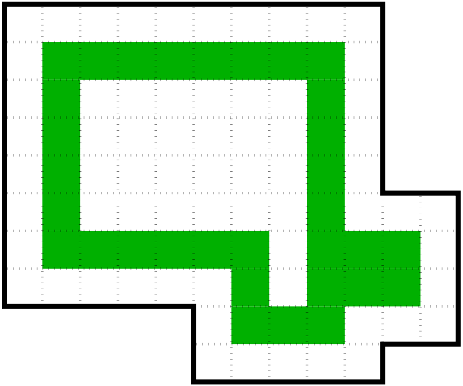
Layer and Offset



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- *1-offset* $:=$
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- Analogously: *Second layer*
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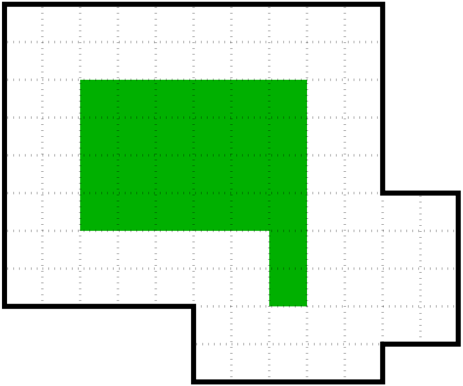
- and so on
- E: #edges between free and blocked cells

Lemma (Number of edges)

P' *is* ℓ*-offset of* $P \Rightarrow E(P') \leq E(P) - 8\ell$.

Layer and Offset



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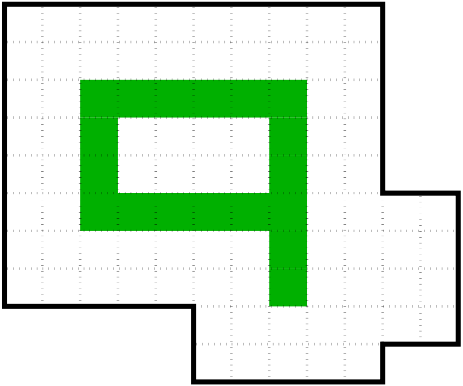

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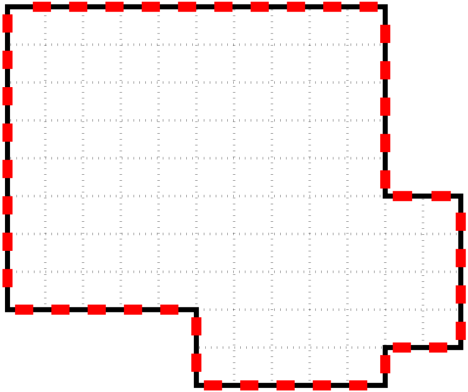

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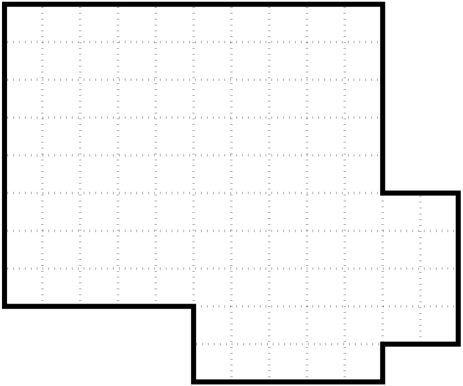

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Shortest path between two cells in $P \leq \frac{1}{2}E(P) - 2$.

Proof sketch.

- Worst case:
  Both cells in the first layer
- $|\pi_{cw}| = |\pi_{ccw}|$
  $= \frac{1}{2} \cdot$ #cells in the first layer
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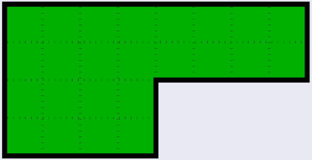

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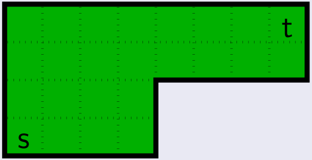

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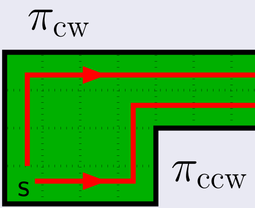

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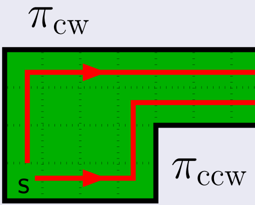

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Upper for the Number of Steps (1)

Theorem (Number of Steps)

$$S \leq C + \frac{1}{2}E - 3 \qquad \textit{(tight!)}$$

(S: #Steps from cell to cell, C: #Cells, E: #Boundary edges)

Proof sketch

- Induction on the number of split cells
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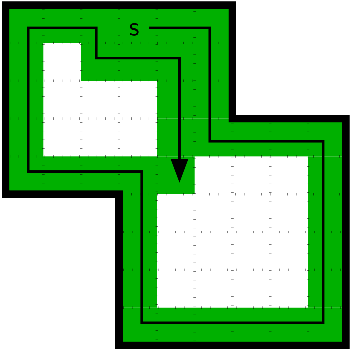

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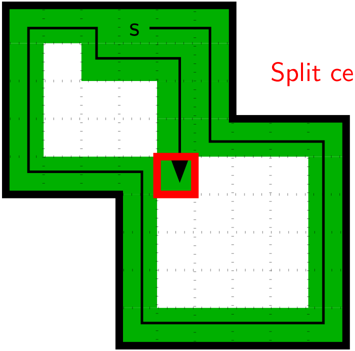

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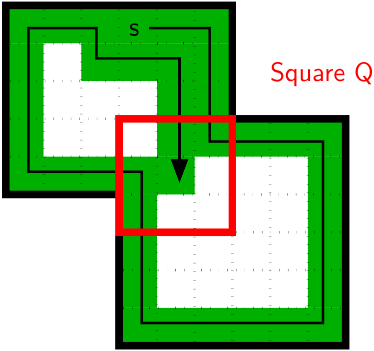

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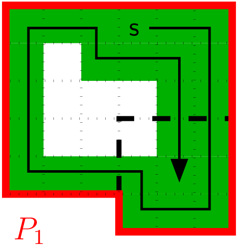

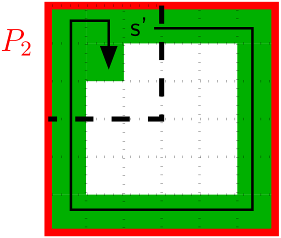

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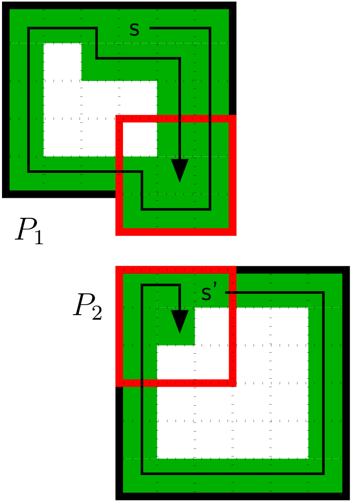

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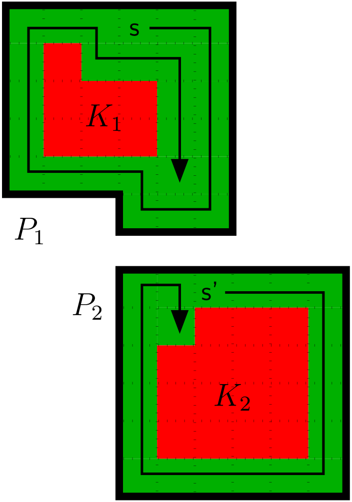

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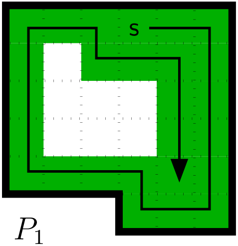

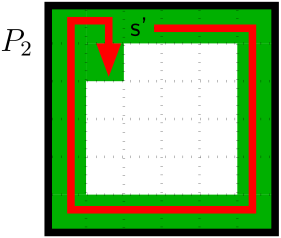

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$$\text{excess}(P) \leq \text{excess}(P_1) + \text{excess}(P_2)$$

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- Apply induction hypothesis to P_1 and K_2

$$\text{excess}(P) \leq \tfrac{1}{2}E(P_1) - 3 + \tfrac{1}{2}E(K_2 \cup \{c\}) - 3 + 1$$

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Theorem (Competitivity)

SmartDFS is $\frac{4}{3}$ competitive (i. e., $S_{\text{SmartDFS}} \leq \frac{4}{3}\, S_{\text{Optimal}}$)

Definition

Narrow passage: Corridors of width 1 or 2.

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Uncritical polygon: neither narrow passages nor split cells in the first layer.

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Competitivity (2)

Lemma (Edges in uncritical polygons)

For uncritical grid polygons: $E(P) \leq \frac{2}{3}C(P) + 6$

Proof.

- Successively remove row or column of at least 3 cells, maintaining the uncritical property
- Ends with 3×3 polygon, $E = \frac{2}{3}C + 6$
- $E \leq \frac{2}{3}C + 6$ fulfilled in every step

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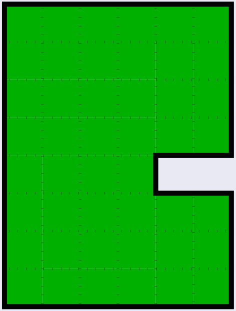

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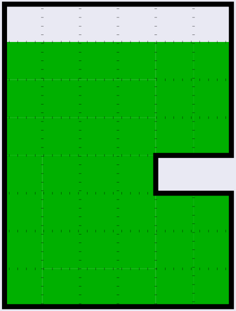

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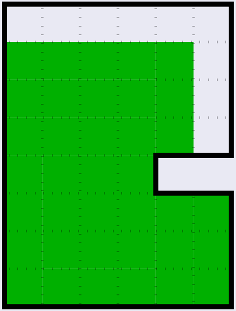

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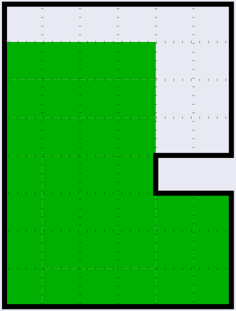

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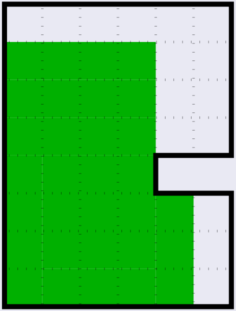

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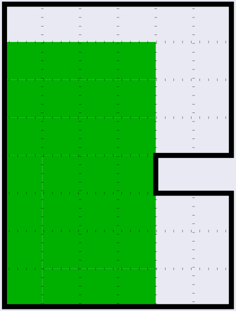

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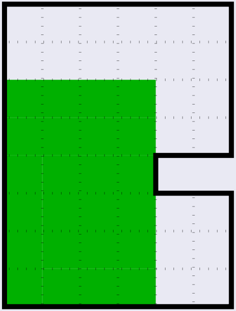

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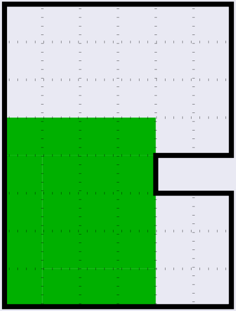

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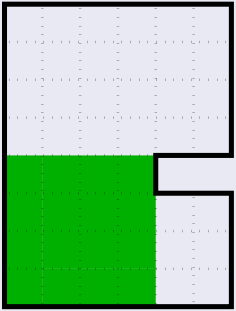

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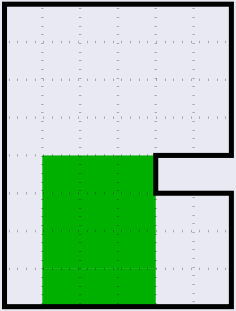

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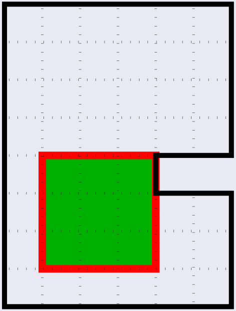

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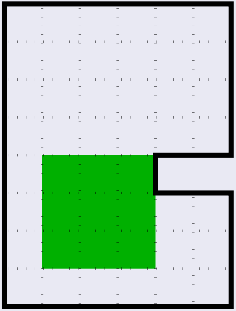

- Successively remove row or column of at least 3 cells, maintaining the uncritical property
- Ends with 3×3 polygon, $E = \frac{2}{3}C + 6$
- $E \leq \frac{2}{3}C + 6$ fulfilled in every step

□

Competitivity (3)

Lemma (Exploration of uncritical polygons)

For uncritical grid polygons: $S(P) \leq C(P) + \frac{1}{2}E(P) - 5$.

Proof sketch

- $S(P) \leq C(P) + \frac{1}{2}E(P) - 3$ shown
- Used shortest path lemma ($sp(c, s) \leq \frac{1}{2}E(P) - 2$)
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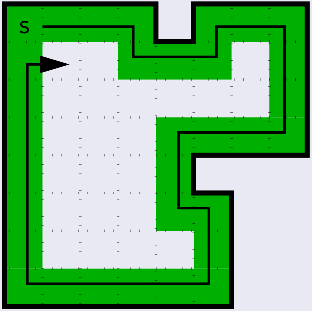

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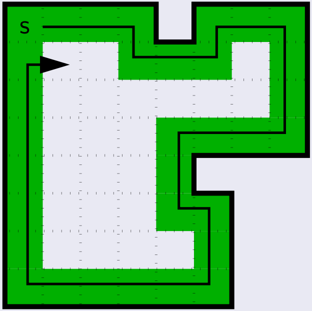

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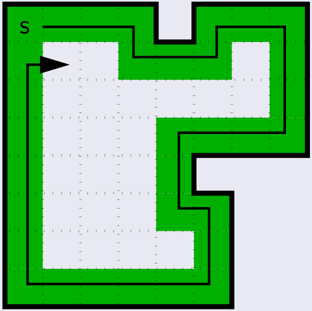

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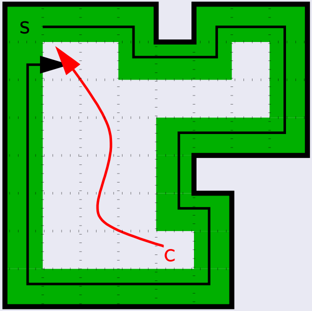

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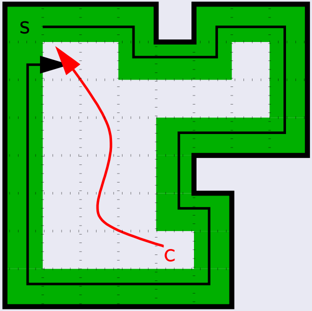

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Theorem (Competitivity)

SmartDFS is $\frac{4}{3}$ competitive.

Proof

- Remove narrow passages (explored optimally)
- $\Rightarrow$ Split P into P_i
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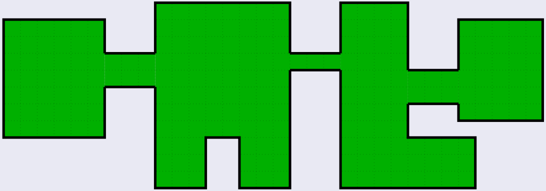

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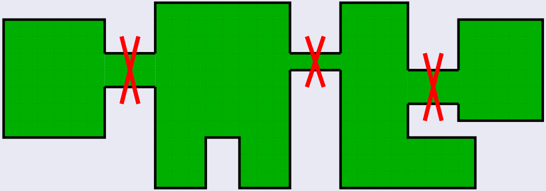

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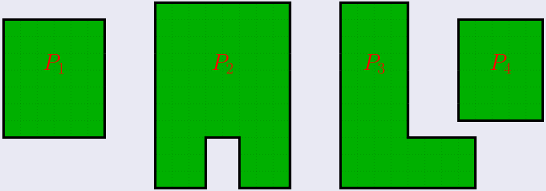

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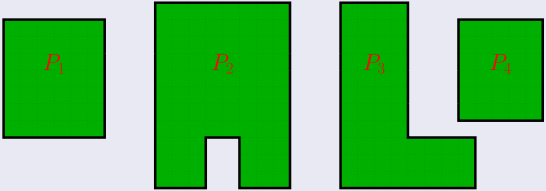

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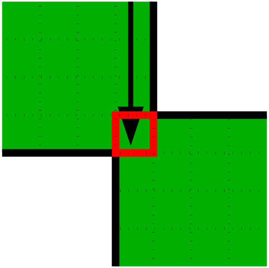

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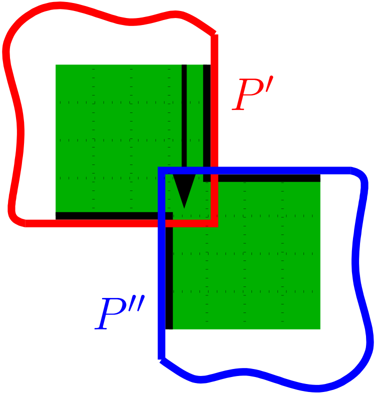

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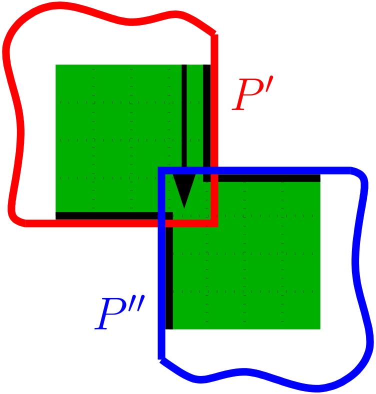

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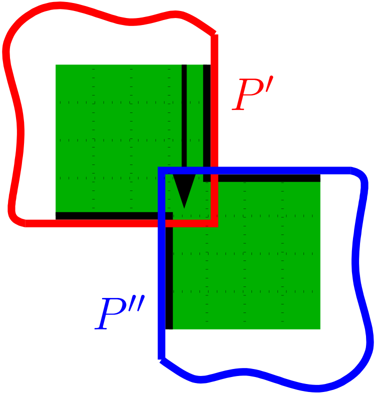

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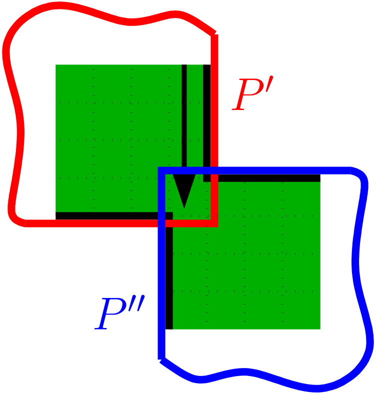

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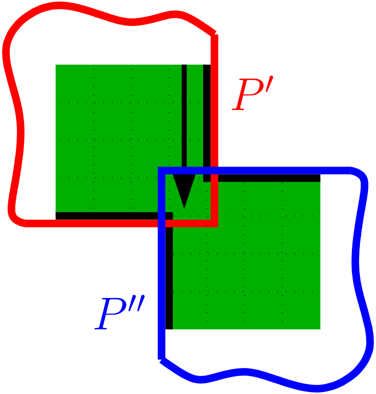

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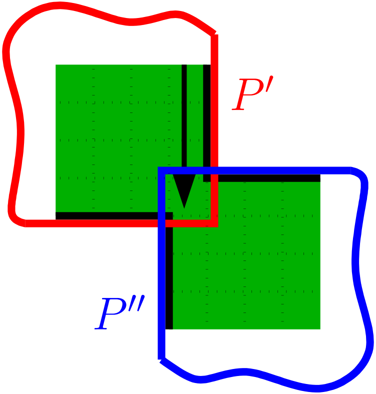

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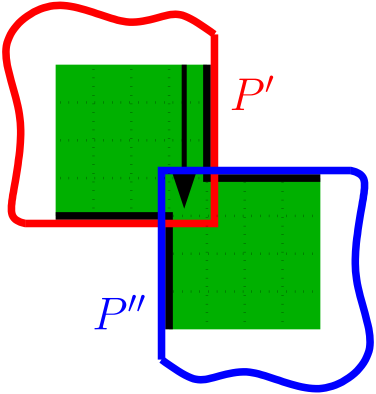

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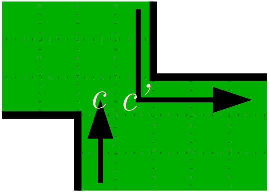
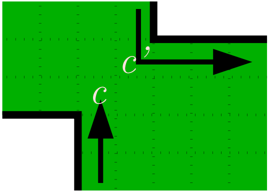
Ind. step, case 2: Robot meets cell c' touching split cell c





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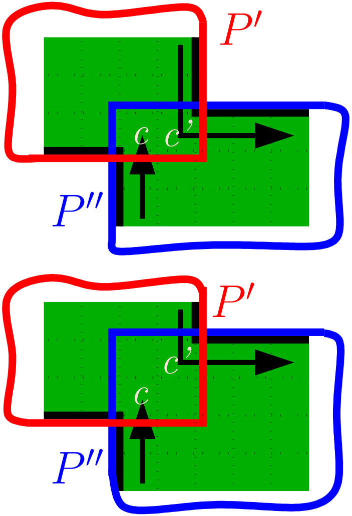

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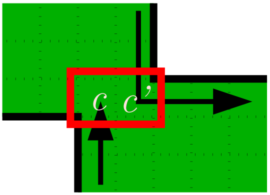

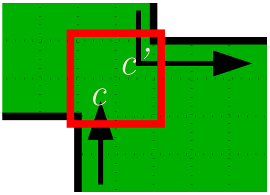

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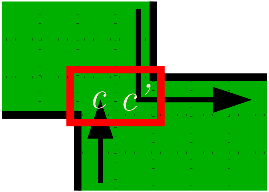

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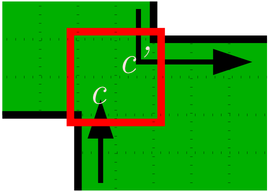

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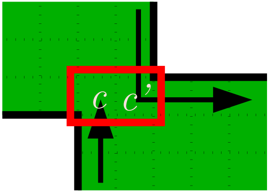

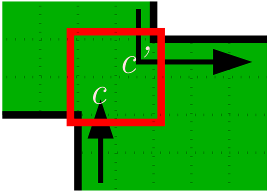

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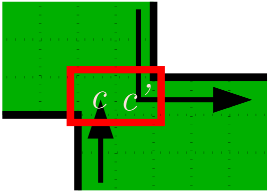

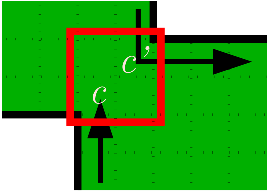

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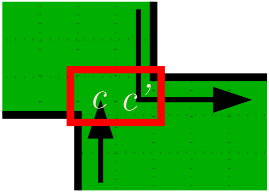

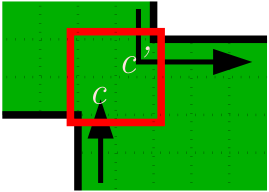

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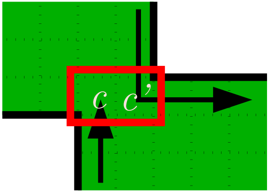
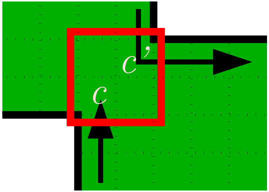
&< \frac{4}{3}C(P_i) - 2 \qquad \square
\end{aligned}
$$

Ind. step, case 2: Robot meets cell c' touching split cell c





- Split P_i into P', P''
- $Q :=$ largest rectangle containing both c, c'
- $C(P_i) = C(P') + C(P'') - |Q|$

$$\begin{aligned} S(P_i) &= S(P') + S(P'') - |Q| \\ &\leq \frac{4}{3}C(P') + \frac{4}{3}C(P'') - 4 - |Q| \\ &= \frac{4}{3}C(P_i) + \frac{1}{3}(|Q| - 6) - 2 \\ &< \frac{4}{3}C(P_i) - 2 \qquad \square \end{aligned}$$

Competitivity Proof (4)

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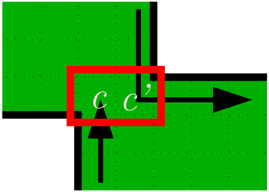

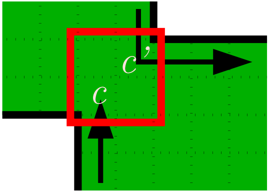

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- Lower Bound: $\frac{7}{6}$
- Exploration strategy *SmartDFS*
- $S \leq C + \frac{1}{2}E - 3$
- $\frac{4}{3}$-competitive

$\longrightarrow$ Accepted for COCOON 2005

- ToDo: Close the gap!

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Thank you!