Exploring Simple Triangular and Hexagonal Grid Polygons Online

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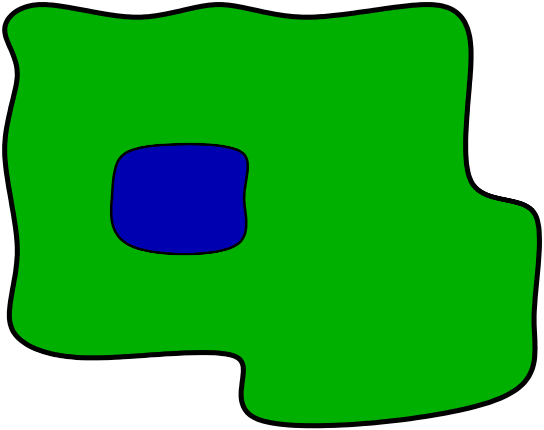
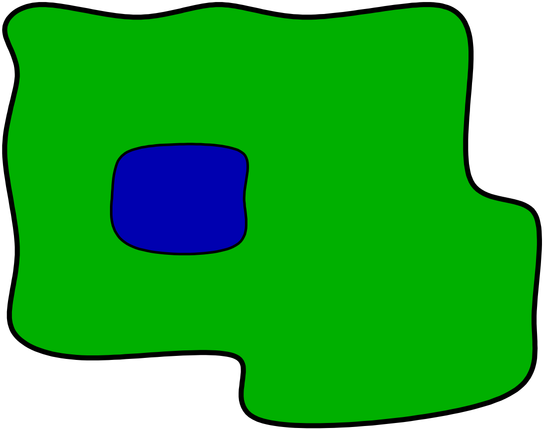


Environment

- Convenient for motion planning tasks:

  Subdivide env. by integer grid
- E.g.: cell size $\approx$ size of robot's tool
- Simple $\Leftrightarrow$ No holes

Grid Environments



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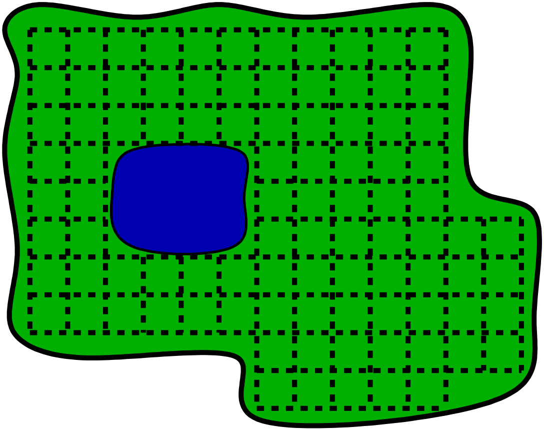

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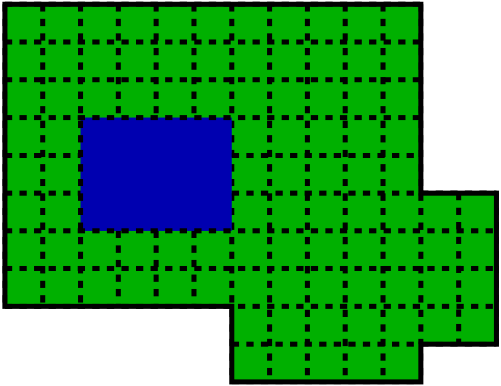

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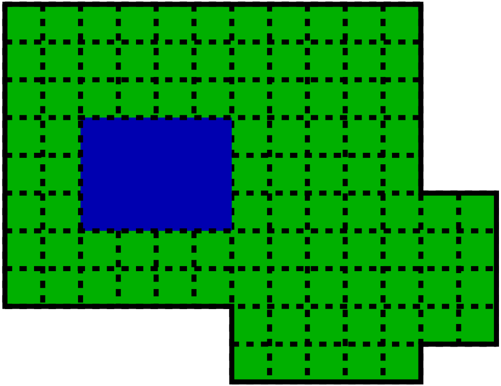

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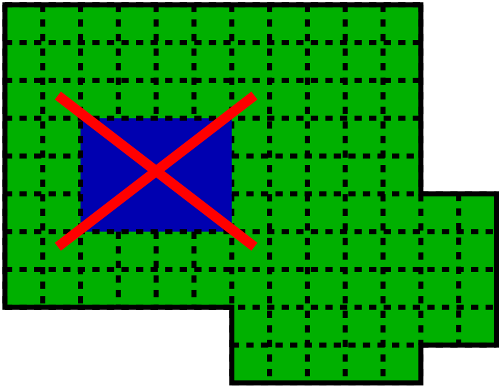

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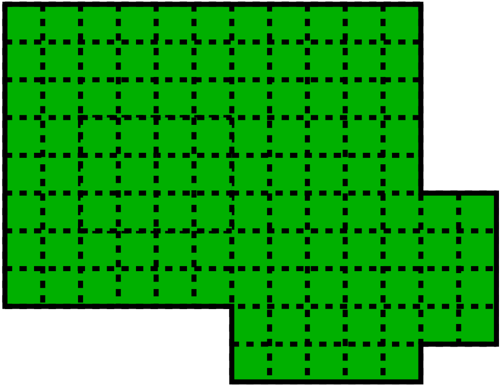

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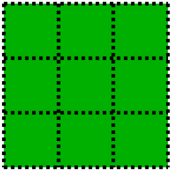
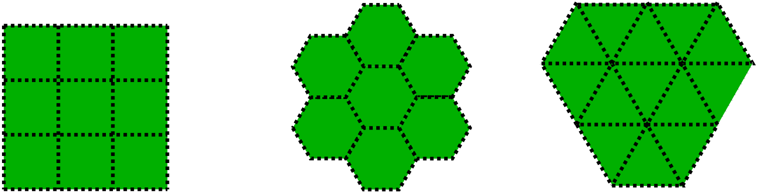


- Usually: Square grids
- Other regular tilings: hexagonal / triangular grids
- Agent:
  - No vision
  - Sense adjacent cells
  - Move to free, adjacent cell



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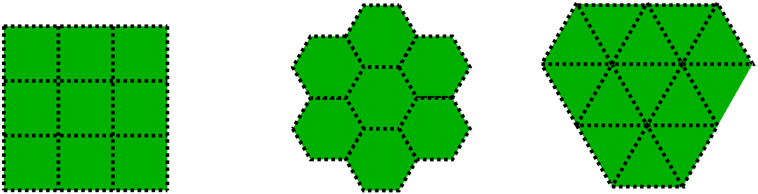

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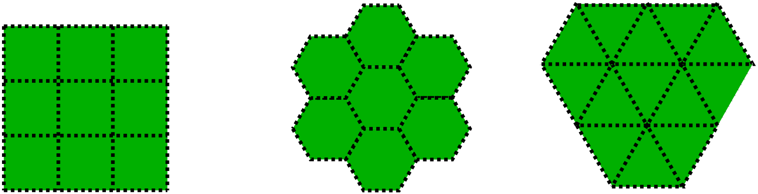

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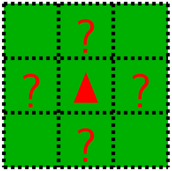

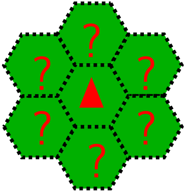

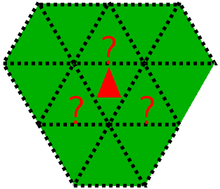

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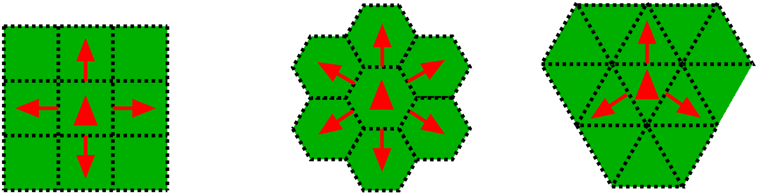

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The Task

Online exploration (or covering):

- Given: an unknown grid environment, P
  start cell, s, along the boundary
- Task: Find a tour that
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  - is as short as possible
- For example: lawn mowing, cleaning

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Previous Work (square grids)

Offline exploration (environment is known in advance)

- With holes: NP-hard [Itai, Papadimitriou, Szwarcfiter; 1982]
- Approx. [Ntafos; 1992] [Arkin, Fekete, Mitchell; 2000]

Online exploration

- With holes:
  [Icking, Kamphans, Klein, Langetepe; 2000]: 2-competitive
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- Without holes:
  [Icking, Kamphans, Klein, Langetepe; 2005]: $\frac{4}{3}$-competitive

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Problem:

Online exploration of simple hexagonal/ triangular grid polygons

Theorem

No online exploration strategy achieves a competitive factor better than

- $\frac{7}{6}$ *in simple* ***triangular*** *grid polygon.*
- $\frac{14}{13}$ *in simple* ***hexagonal*** *grid polygon.*

Lower bound for polygons with holes: 2 [IKKL 2000]

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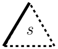
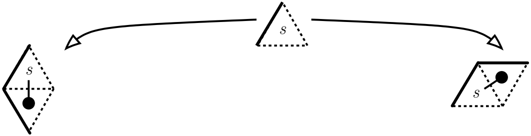
Proof: Lower Bound (Triangular Grids)



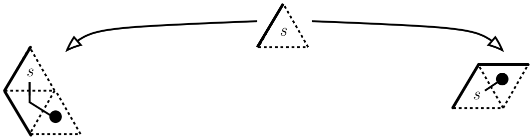
Proof: Lower Bound (Triangular Grids)

South or East



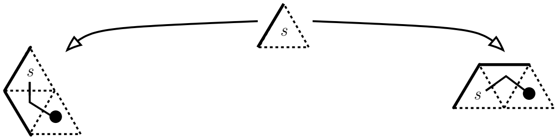
Proof: Lower Bound (Triangular Grids)

East



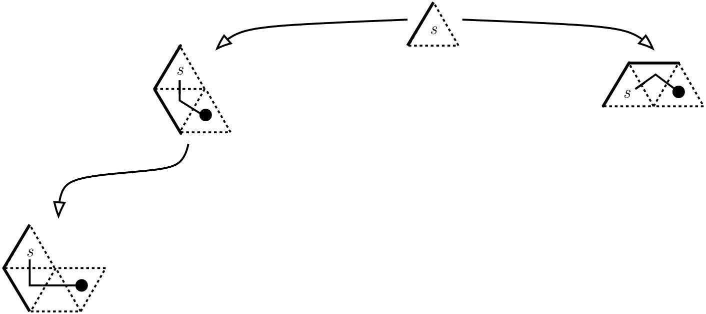
Proof: Lower Bound (Triangular Grids)

East



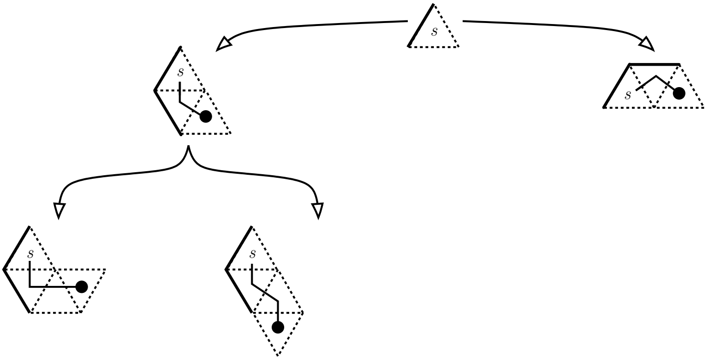
Proof: Lower Bound (Triangular Grids)

2 Possibilities: South



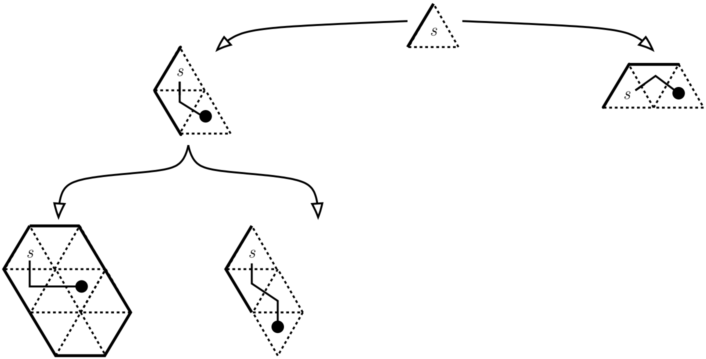
Proof: Lower Bound (Triangular Grids)

2 Possibilities: South, East



Proof: Lower Bound (Triangular Grids)

Close polygon



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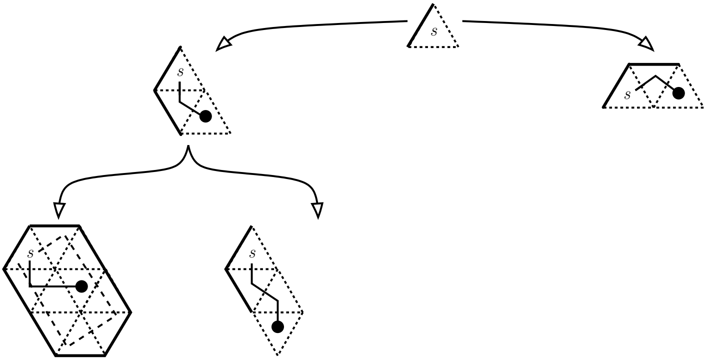
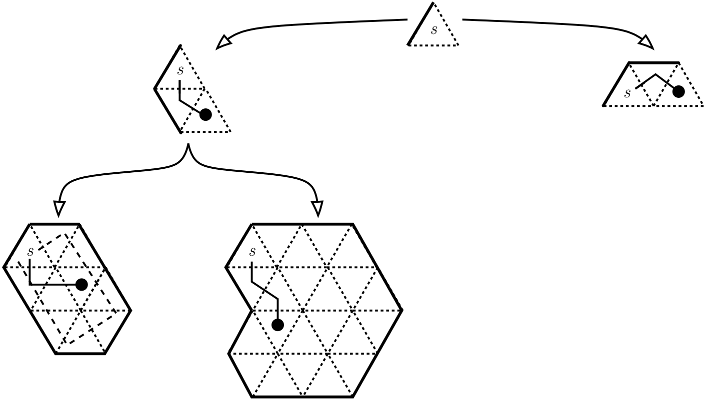
Online vs. Optimal



$\frac{12}{10}$

Close polygon



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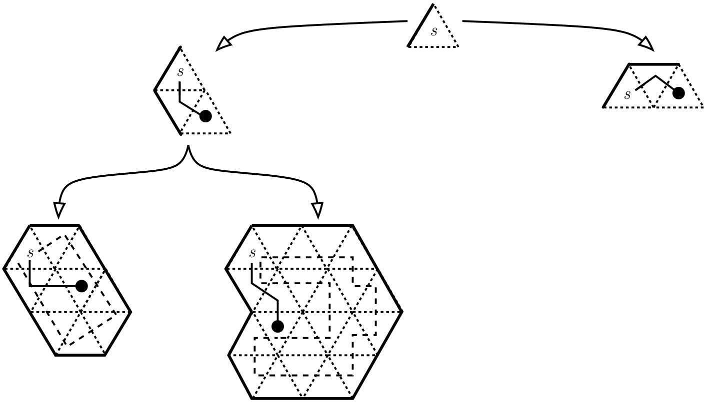

$\frac{12}{10}$ $\frac{26}{22}$

Proof: Lower Bound (Triangular Grids)

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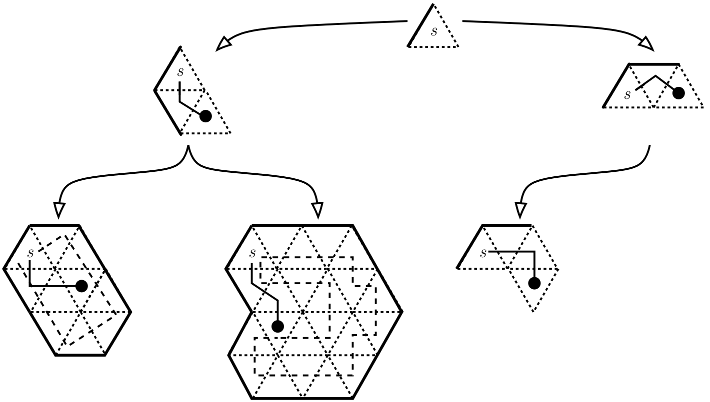

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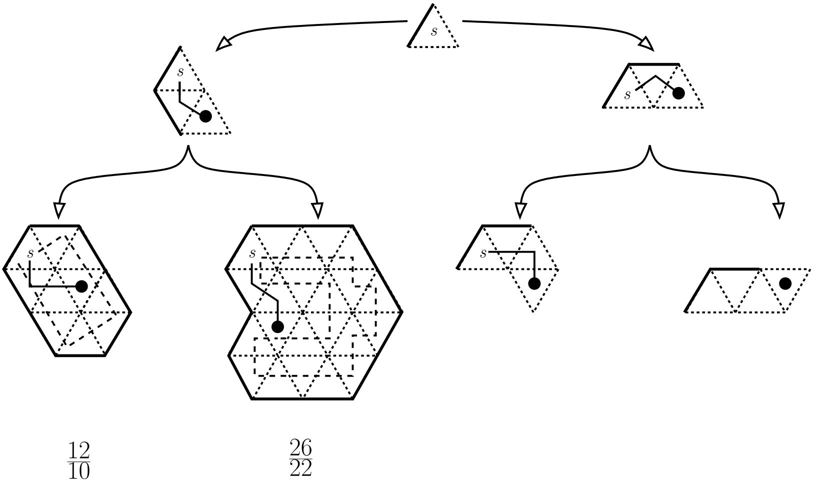

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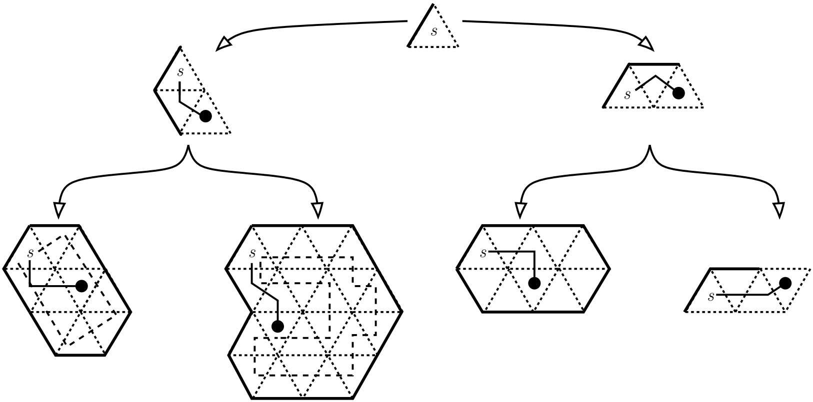

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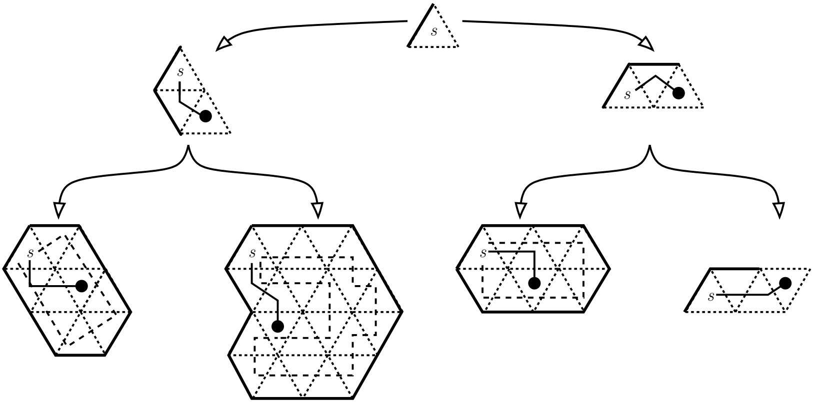

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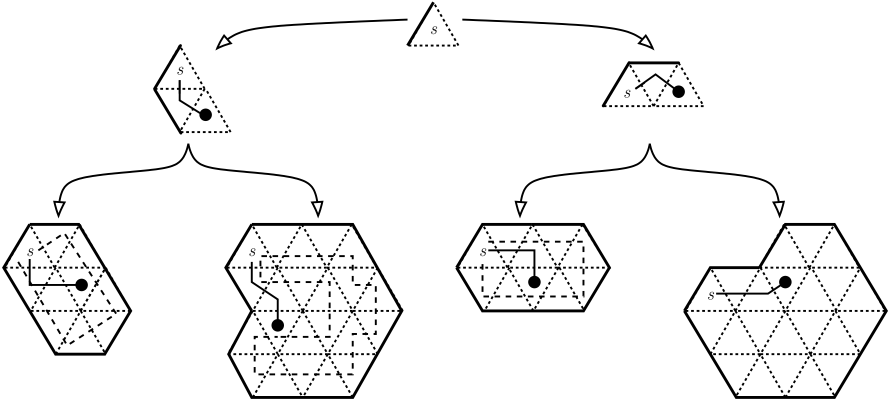

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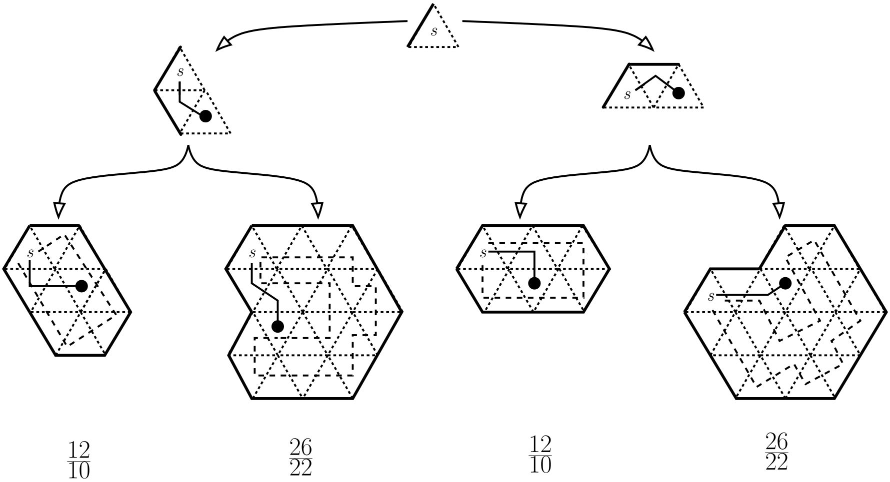
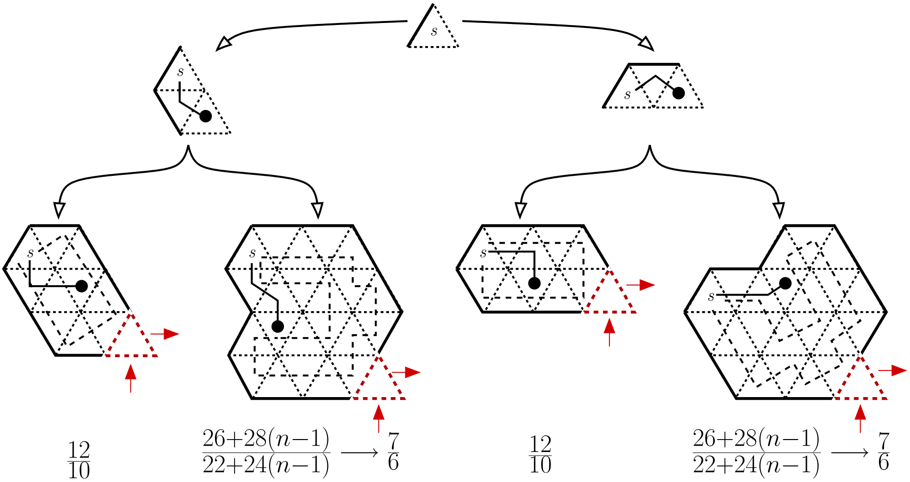
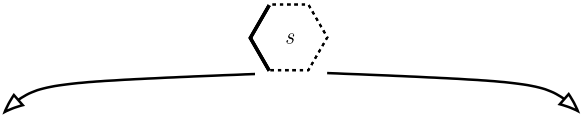


Proof: Lower Bound (Triangular Grids)

Polygons of arbitrary size



Proof: Lower Bound (Hexagonal Grids)



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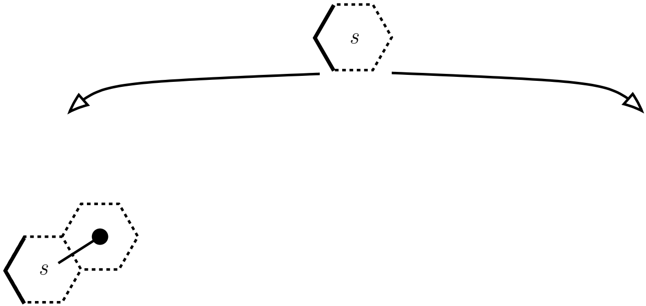
Leave boundary



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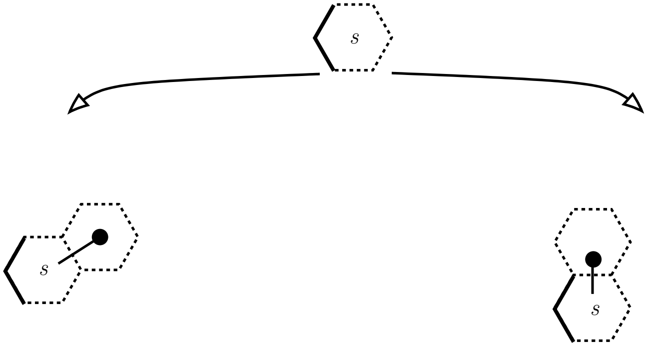
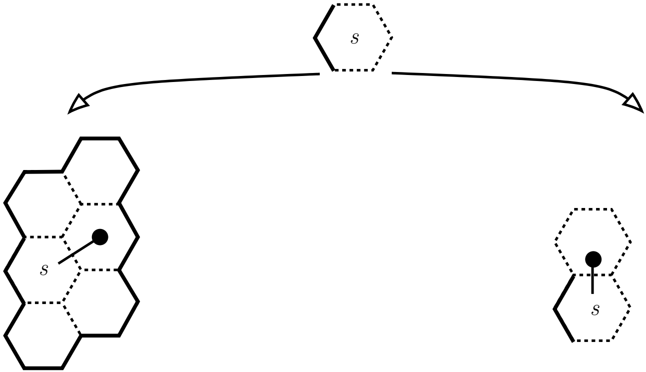
Follow boundary



Proof: Lower Bound (Hexagonal Grids)

Close block



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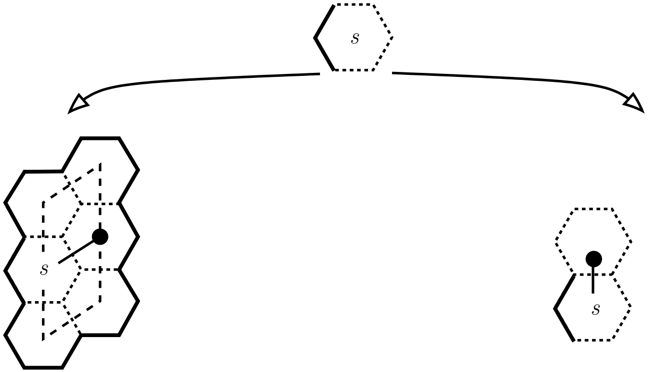
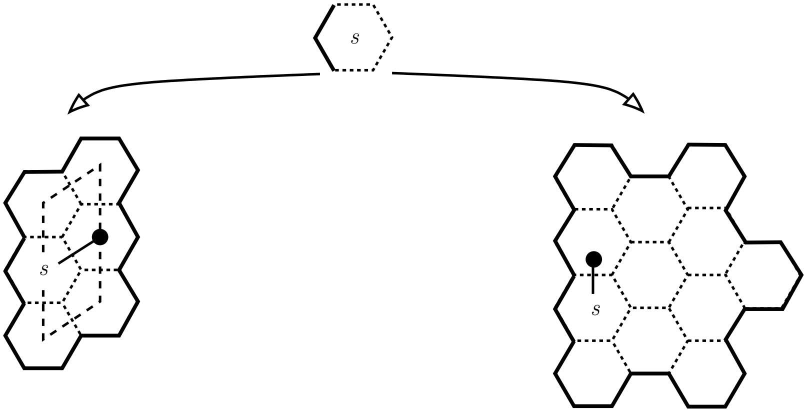


$$\frac{7}{6}$$

Close block



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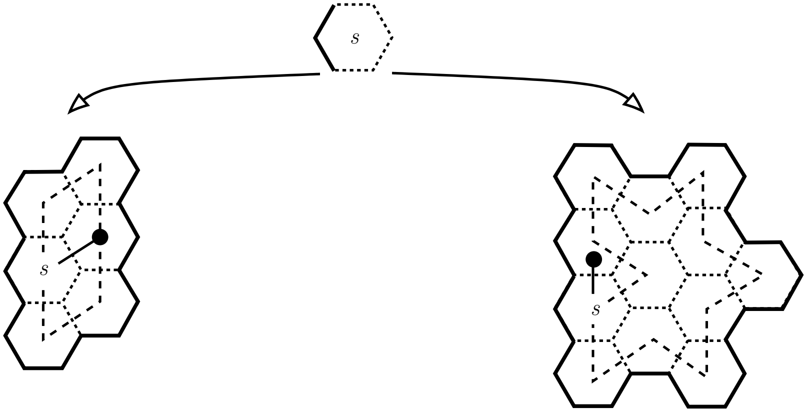

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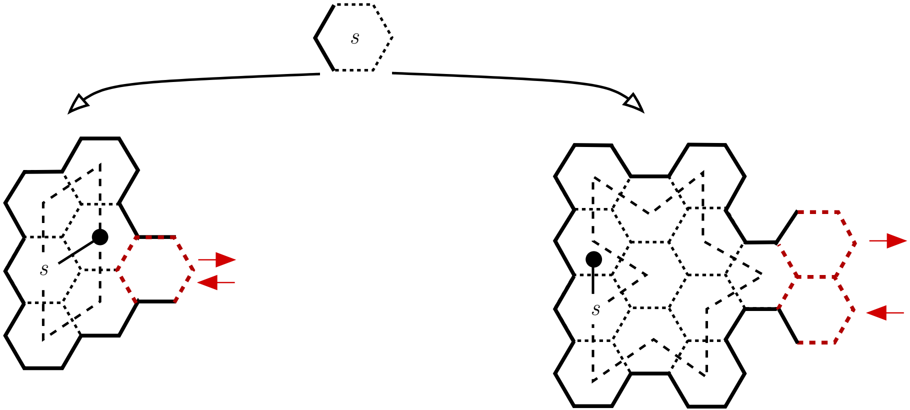

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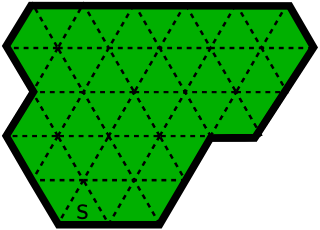
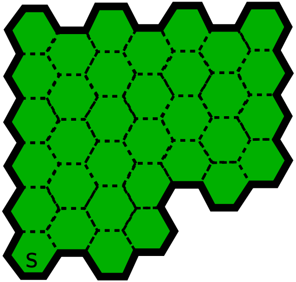
$$\frac{13+14(n-1)}{12+13(n-1)} \longrightarrow \frac{14}{13}$$

SmartDFS: An exploration strategy (1)





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- Visits *each* cell twice!

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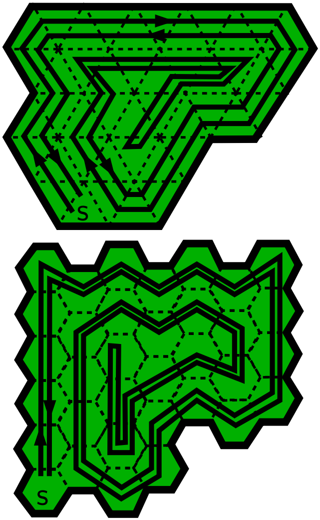

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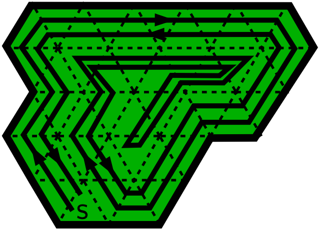

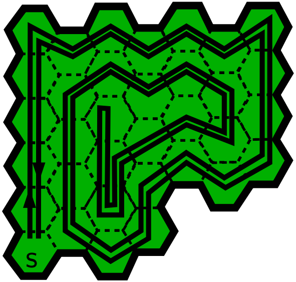

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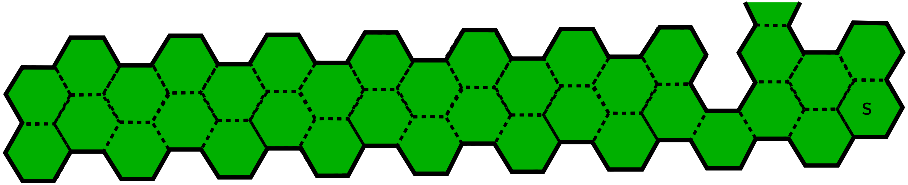

- DFS visits each cell twice
- More reasonable: Return directly to unvisited cell
- Improved DFS

Improvement 1

Return directly to those cells that have unexplored neighbors.

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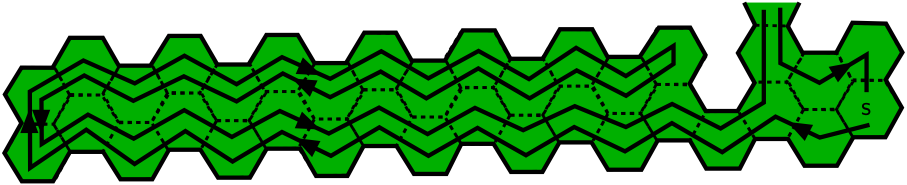

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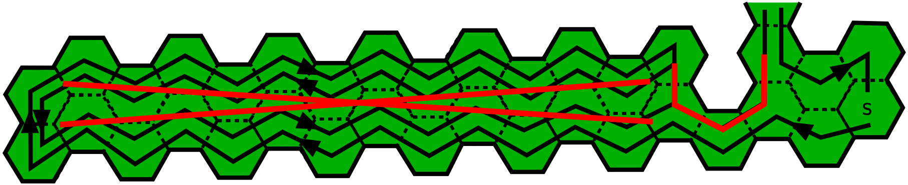

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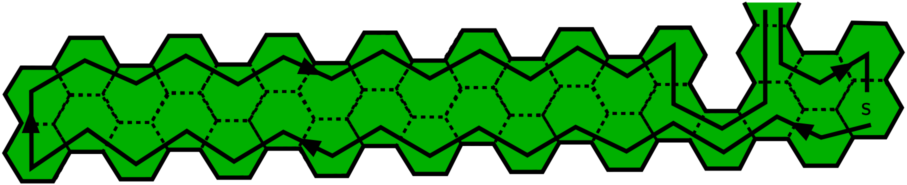

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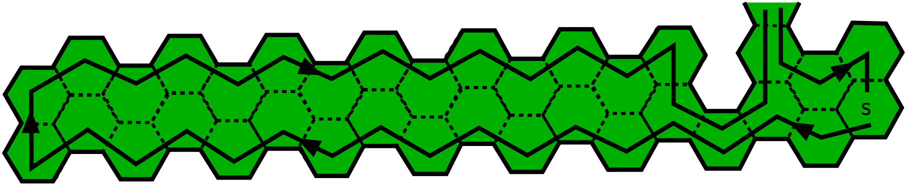

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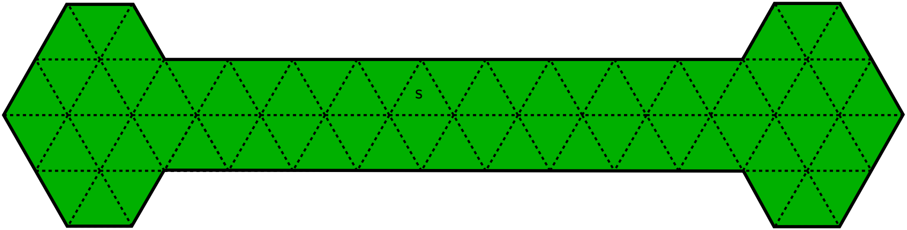

- DFS visits long corridor four times
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Detect and handle split cells (i. e., prefer parts of *P* farther away from the start).

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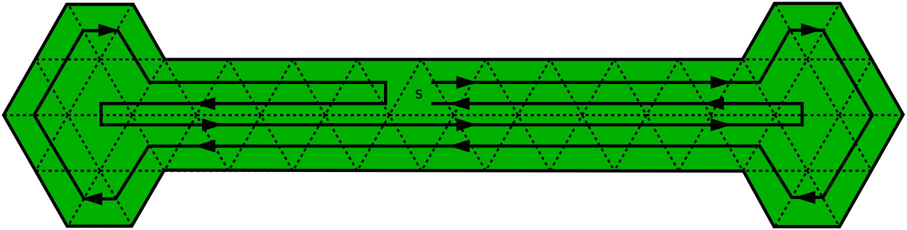

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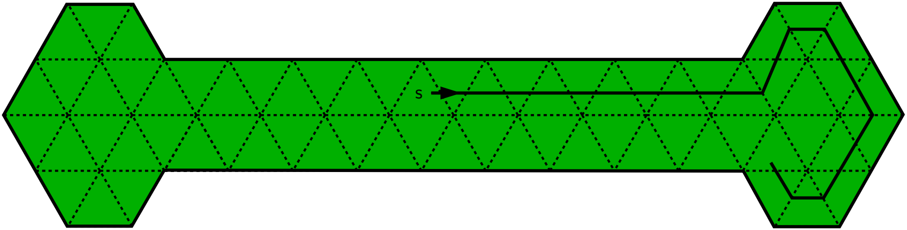

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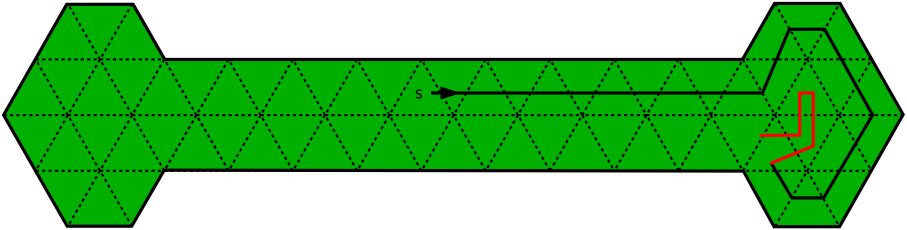

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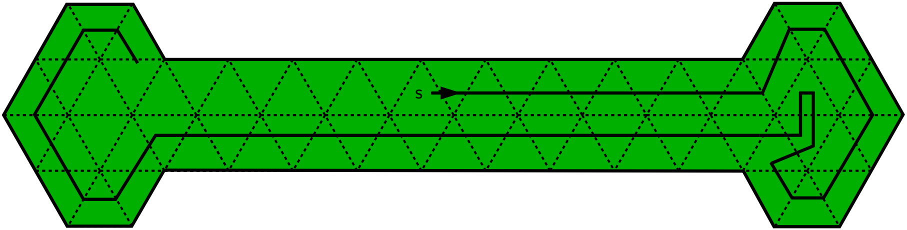

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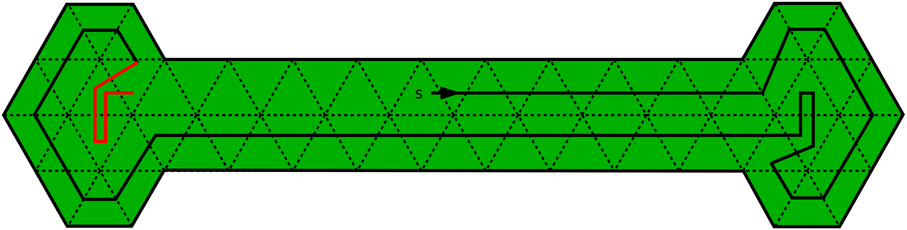

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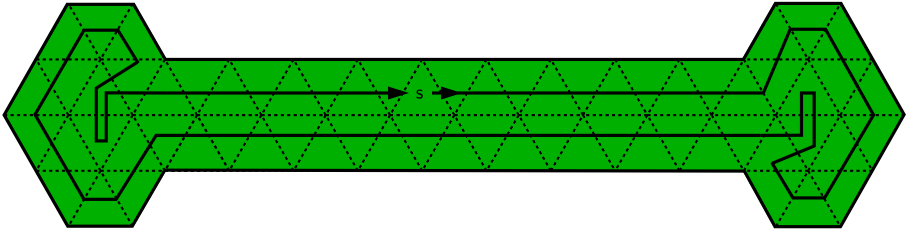

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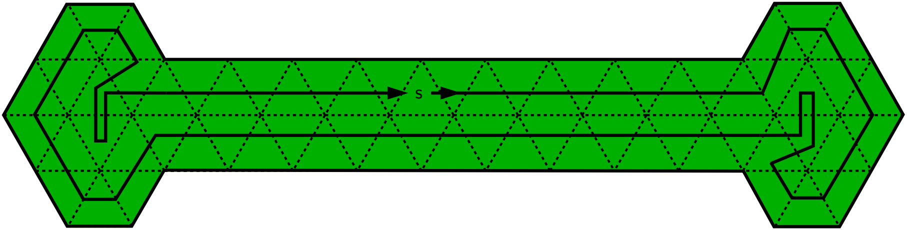

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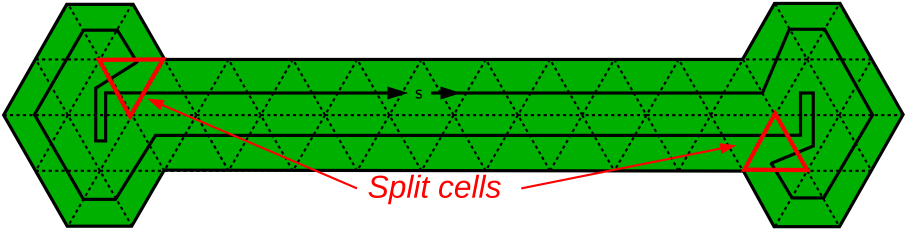

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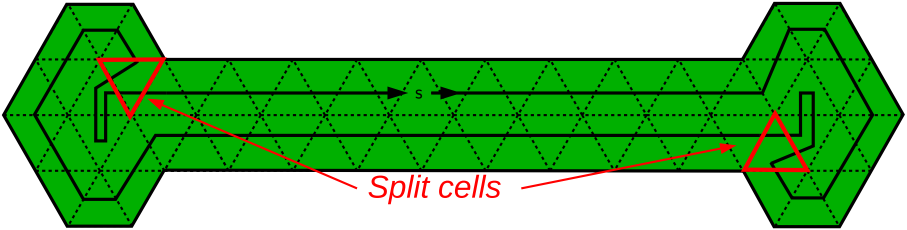

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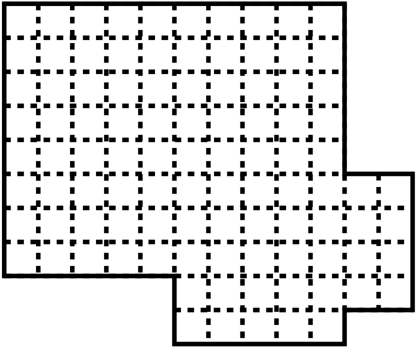

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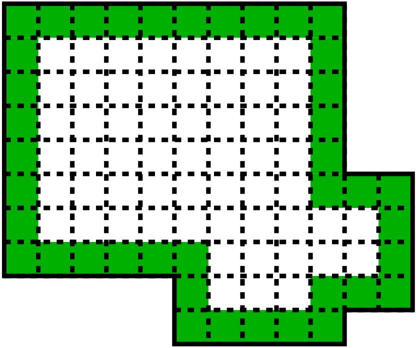
Lemma (Number of edges in offsets)

P' is ℓ-offset of $P \Rightarrow E(P') \leq E(P) - 2k\ell$

($k \in \{3, 4, 6\}$ for $\triangle$, $\square$, $\hexagon$).



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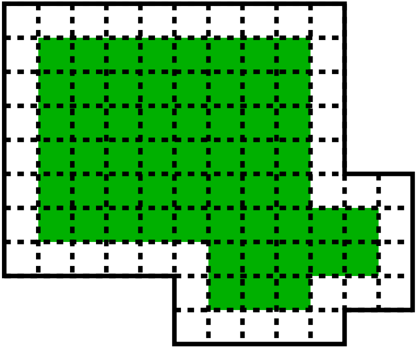

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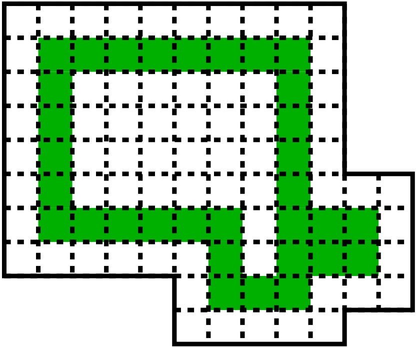

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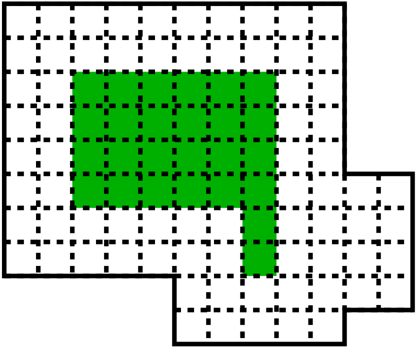

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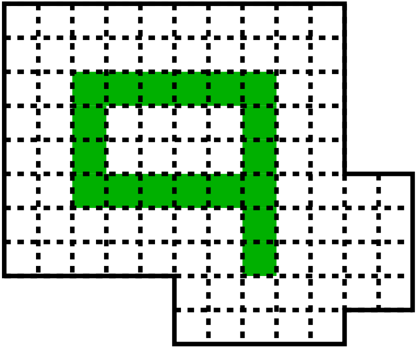

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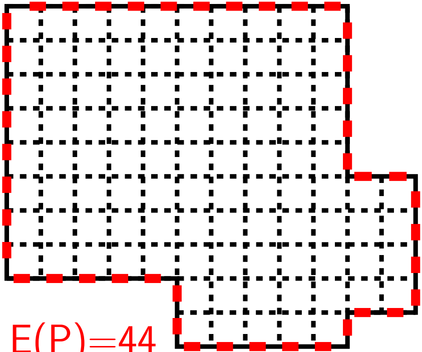

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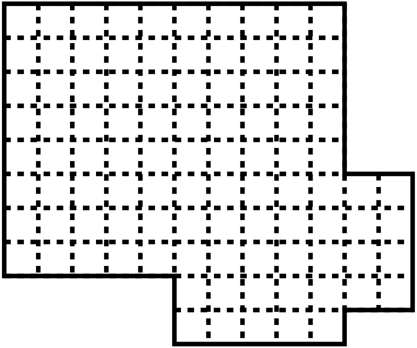

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Shortest path between two cells in P:

- $sp(s,t) \leq E(P) - 3$ *(triangular grids)*
- $sp(s,t) \leq \frac{1}{4}E(P) - \frac{3}{2}$ *(hexagonal grids)*

Proof idea.

- Worst case: $\Rightarrow s, t$ in the first layer
  $\Rightarrow$ Path length $\leq \frac{1}{2} \cdot \#$cells in the first layer
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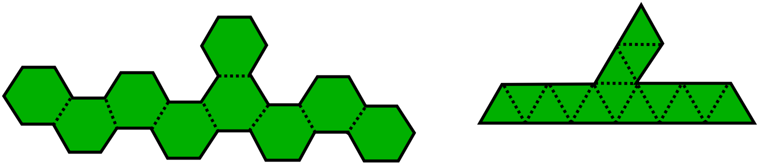
Theorem (Number of Steps)

$$S(P) \leq C(P) + E(P) - 4 \qquad \textit{(Triangular grids)}$$

$$S(P) \leq C(P) + \frac{1}{4}E(P) - \frac{5}{2} \qquad \textit{(Hexagonal grids)}$$

($S(P)$: #Steps from cell to cell, $C(P)$: #Cells, $E(P)$: #Boundary edges)

This bound is exactly achieved in corridors of width 1.



Proof sketch for triangular grids

- $S(P) = \underbrace{C(P)}_{\text{Cells, i.e., necessary steps}} + \underbrace{ex(P)}_{\text{additional cell visits}}$
- Show: $ex(P) \leq E(P) - 4$ (triang.)
- Induction on the number of split cells
- Induction base: No split cell
- Visit every cell in $C(P) - 1$ steps
- Return to s in $\leq E(P) - 3$ steps (Shortest-path lemma)

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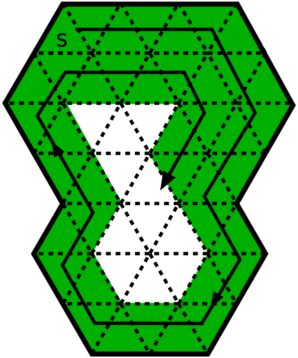

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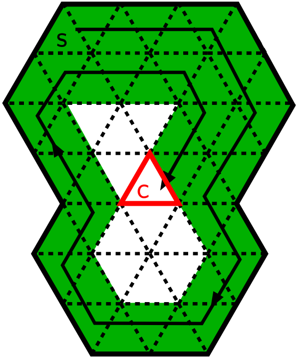

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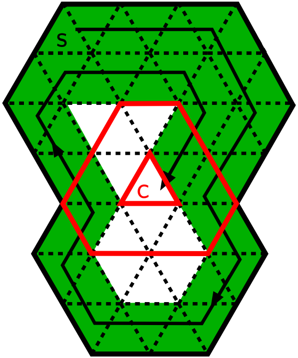

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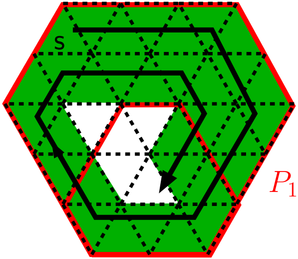

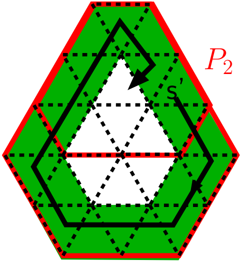

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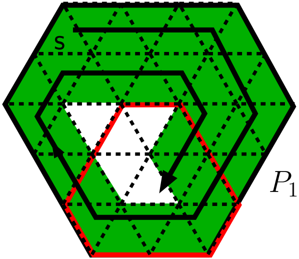

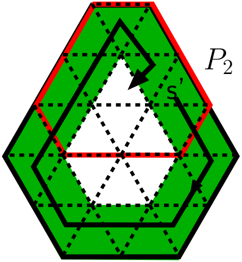

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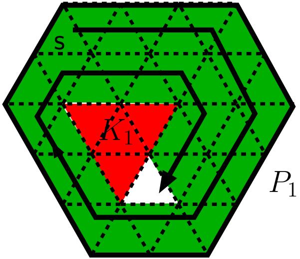

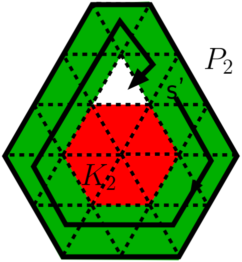

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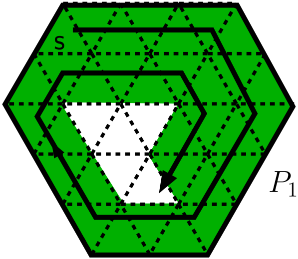

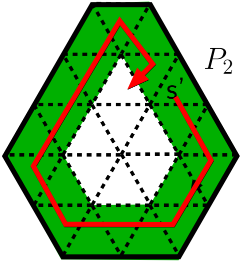

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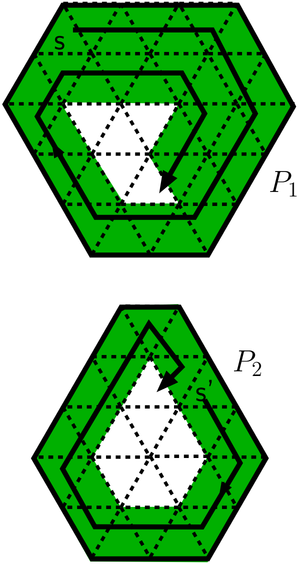

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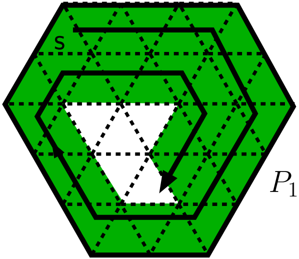

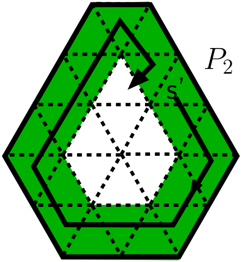

Induction step:

- Explore up to the first split cell, c
- Enlarge c to boundary (add layers)
- Split polygon in two parts: P_1, P_2
- Path outside Q do not change
- K_2 unexplored part of P_2
- Path in $P_2 \setminus K_2$ visits no cell twice
- Apply induction hypothesis to P_1 and $K_2 \cup \{c\}$
- Charge edges of P_1, K_2, Q to $E(P)$

$\square$

Competitivity, hexagonal and triangular grids

Theorem (Competitivity)

SmartDFS is $\frac{4}{3}$ competitive (i. e., $S_{\text{SmartDFS}} \leq \frac{4}{3}\, S_{\text{Optimal}}$)

Definition

Narrow passage: Corridors of width 1 or 2.

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For uncritical grid polygons: $E(P) \leq \frac{1}{3}C(P) + \frac{14}{3}$

Proof idea.

- Successively remove straight line of at least 3 cells (thus at most 1 edge), maintaining the property 'uncritical'
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Competitivity (3)

Lemma (Exploration of uncritical polygons)

For uncritical triangular grid polygons: $S(P) \leq C(P) + E(P) - 6$.

Proof sketch

- Shown: $S(P) \leq C(P) + E(P) - 4$
- Used shortest path lemma: $sp(c, s) \leq E(P) - 4$
- Proof assumed c, s in the first layer!
- Now: c in the 1-offset
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Proof

- Remove narrow passages (explored optimally)
- $\Rightarrow$ Split P into P_i
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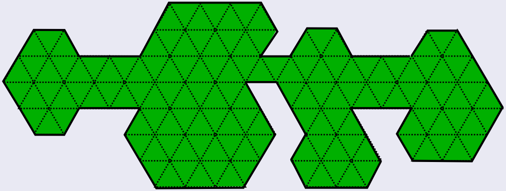

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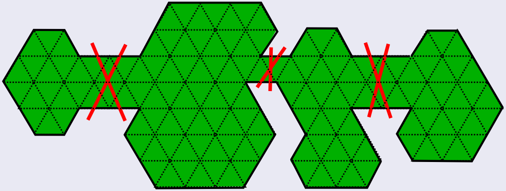

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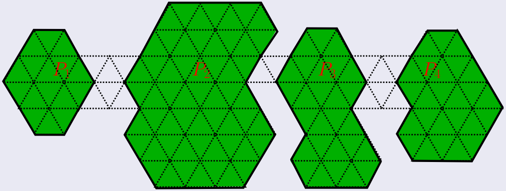

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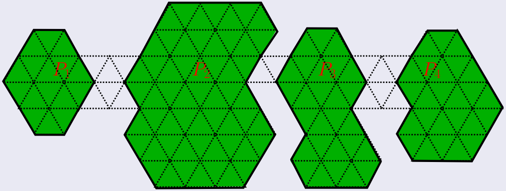

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- Show $S(P_i) \leq \frac{4}{3}C(P_i) - \frac{4}{3}$ by induction on the number of split cells in the first layer
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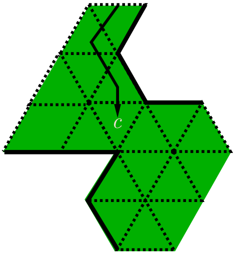

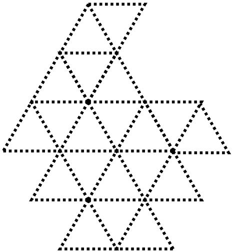

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- $R := \{c\}$
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  - $C(P_i) = C(P') + C(P'') - |R|$
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  - Apply induction hypothesis to $S(P')$, $S(P'')$ □
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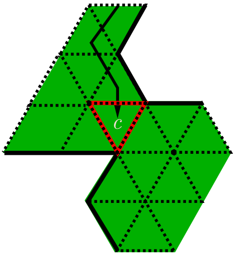

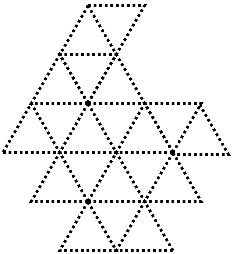

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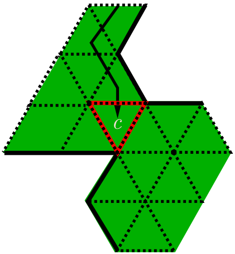

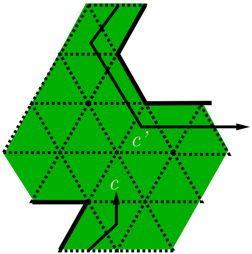

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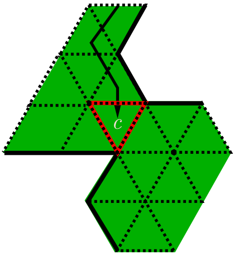

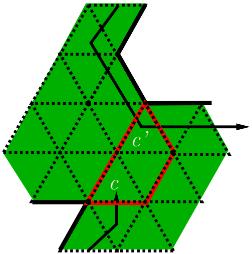

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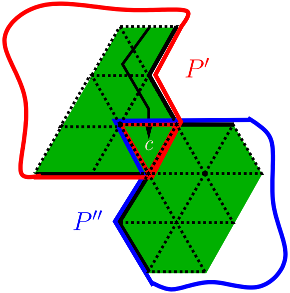

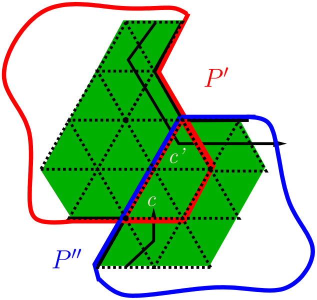

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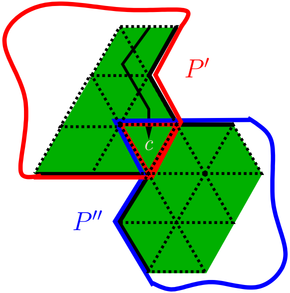

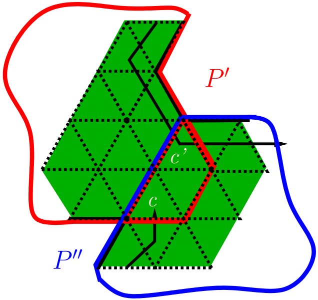

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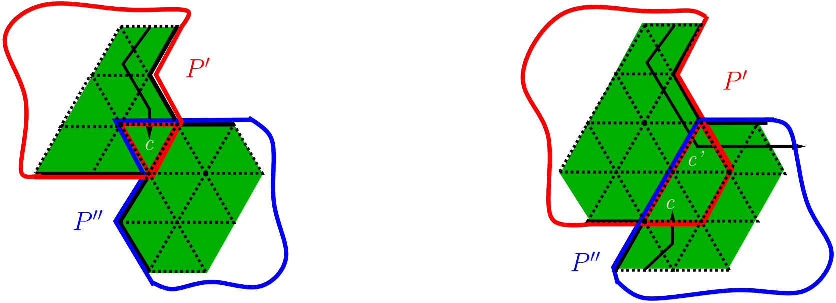

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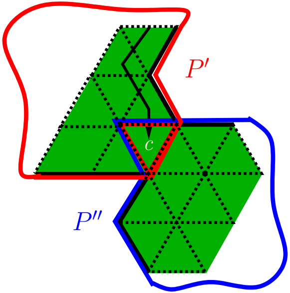

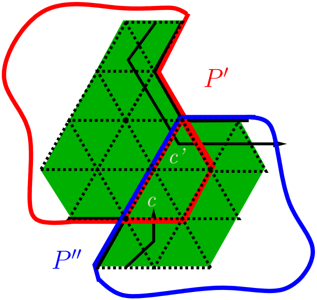

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- Lower bound △: $\frac{7}{6}$
- Lower bound ⬡: $\frac{14}{13}$
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Thank you!