Exploring Simple Grid Polygons

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COCOON 2005

- Robot, R, has to explore an unknown environment, P
- More precisely, find a tour that
  - visits every part of P at least once
  - returns to the robot's start point
  - can be computed online
  - is as short as possible
- For example: lawn mowing, cleaning

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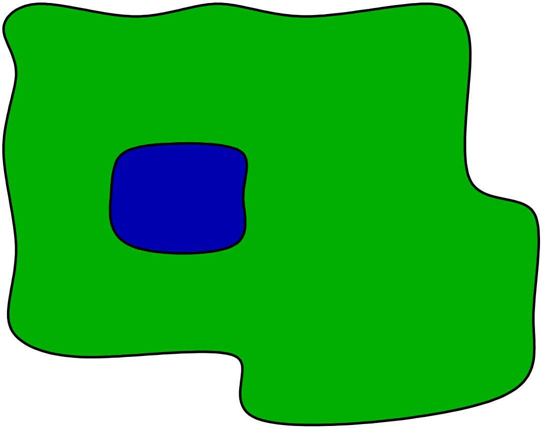
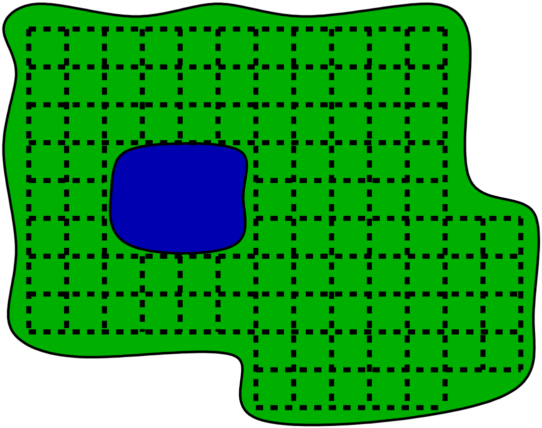


Grid polygon:

- Environment is subdivided by an integer grid
- Simple $\Rightarrow$ No holes

Robot

- No vision
- Can sense 4 adjacent cells
- Can enter adjacent, *free* cell



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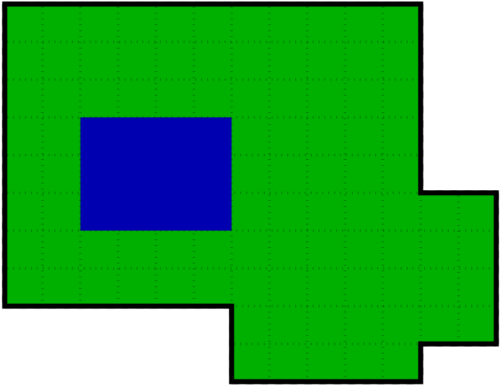

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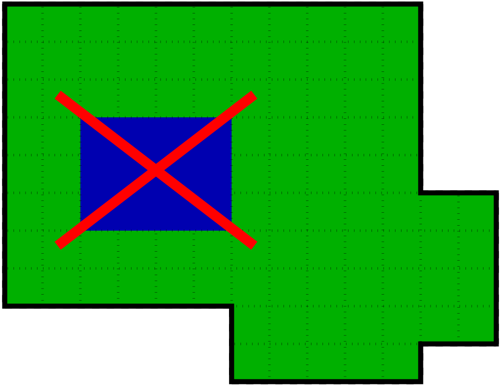

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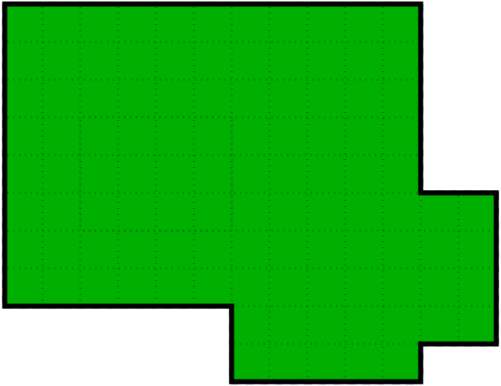

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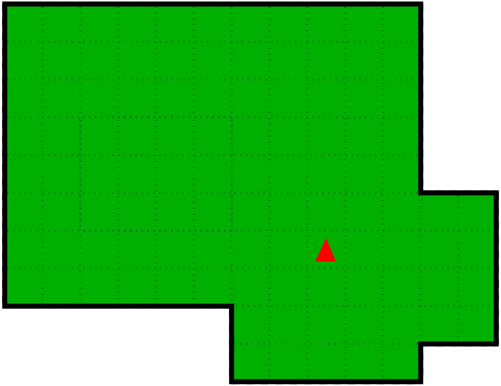

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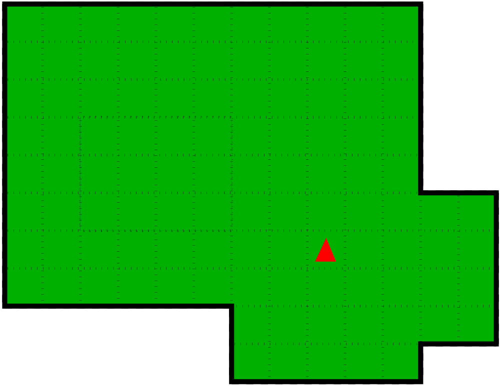

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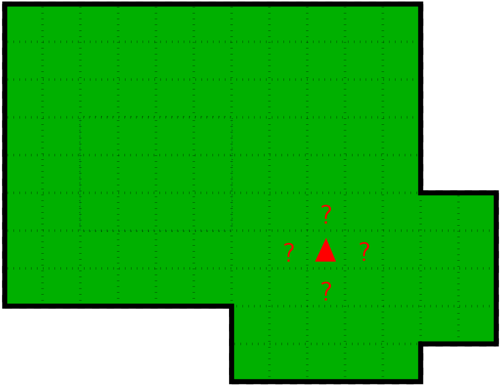

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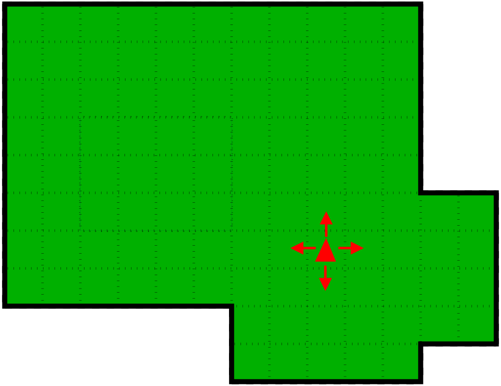

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Offline (i. e., environment is known to the robot)

- With holes:
  NP-hard [Itai, Papadimitriou, Szwarcfiter; 1982]
- Without holes:
  $\frac{4}{3}$-approximation [Ntafos; 1992]
  $\frac{6}{5}$-approximation [Arkin, Fekete, Mitchell; 2000]

Online

- With holes:
  [Icking, Kamphans, Klein, Langetepe; 2000]
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Theorem (IKKL; 2000)

Lower bound on the online exploration of grid polygons with *holes: 2.*

Theorem

There is a $\frac{4}{3}$-competitive online exploration strategy for polygons without *holes.*

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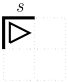
There is a $\frac{4}{3}$*-competitive online exploration strategy for polygons* without *holes*.

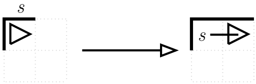
Theorem

No online exploration strategy achieves a factor better than $\frac{7}{6}$ in simple grid polygon.

Proof: Lower Bound

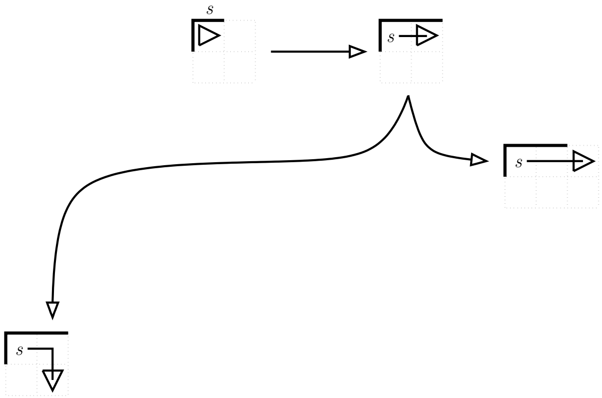


w. l. o. g.: East



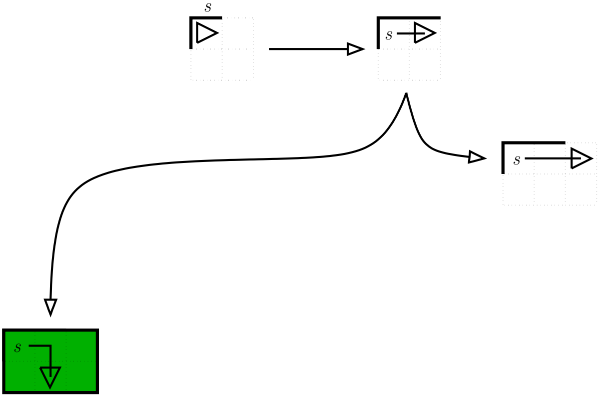
Proof: Lower Bound

South or East

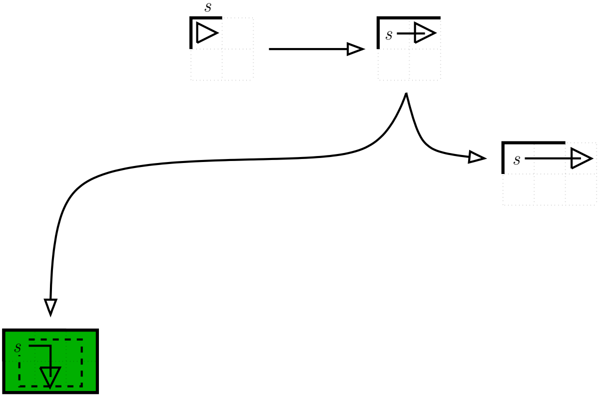


Proof: Lower Bound

Close Polygon



Online vs. Optimal



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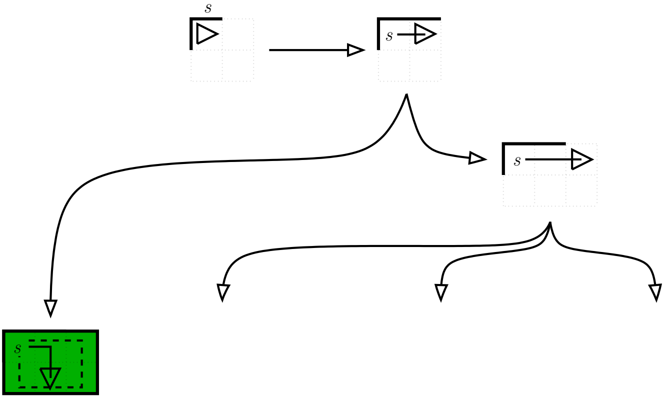
3 Possibilities:



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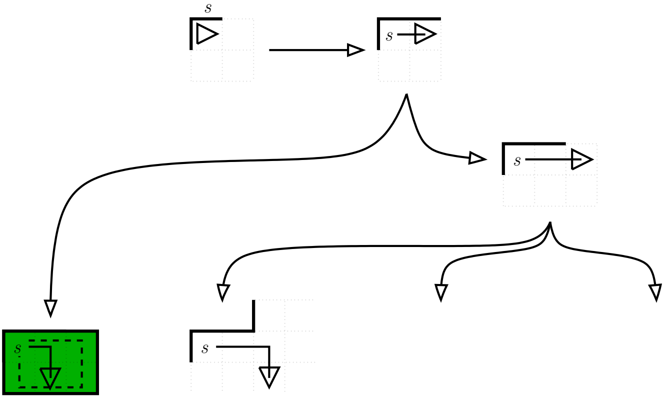
3 Possibilities: South,



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Proof: Lower Bound

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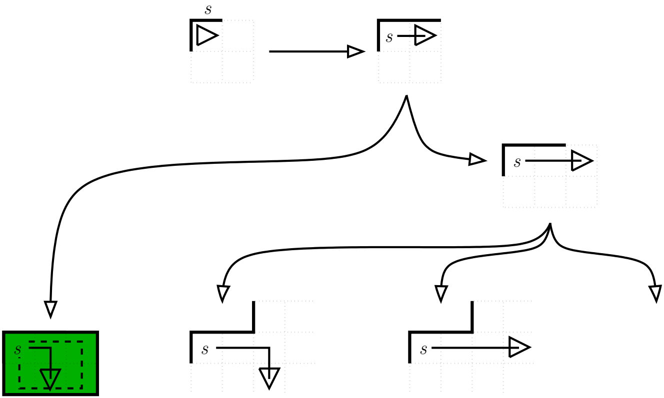

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Proof: Lower Bound

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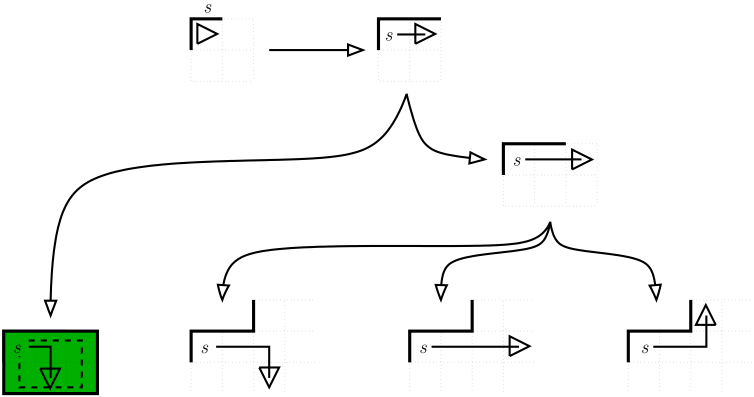

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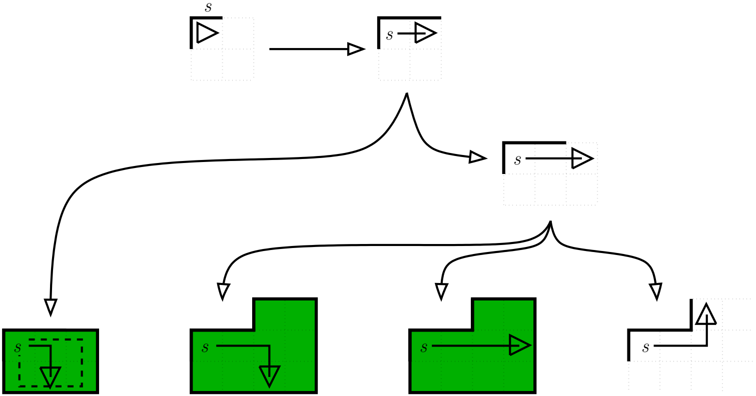

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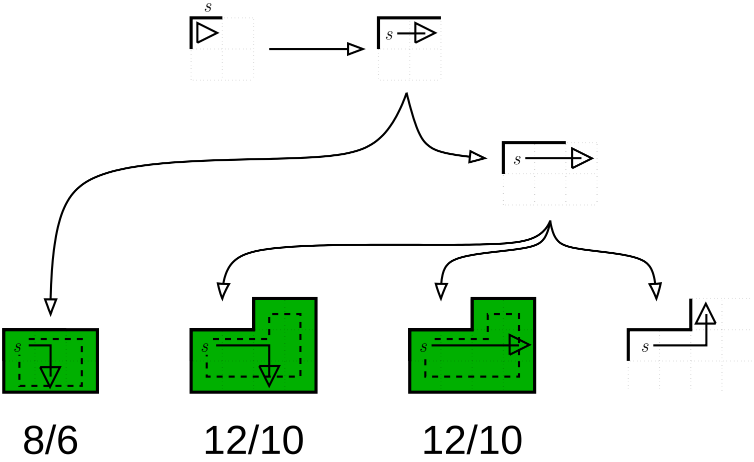

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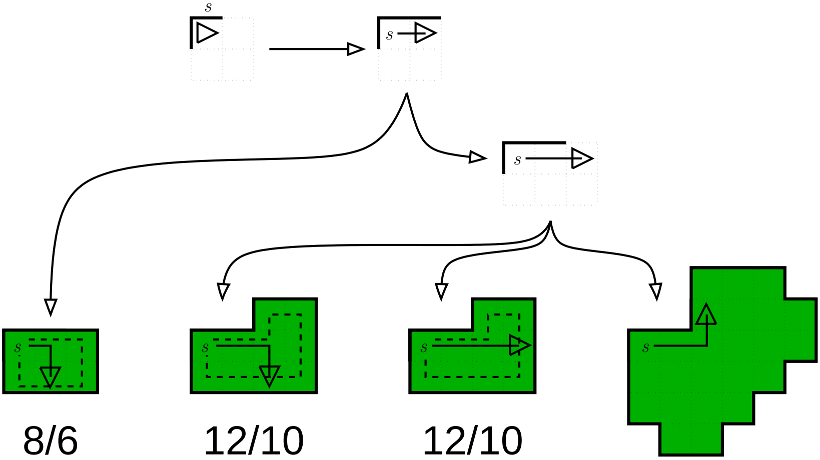

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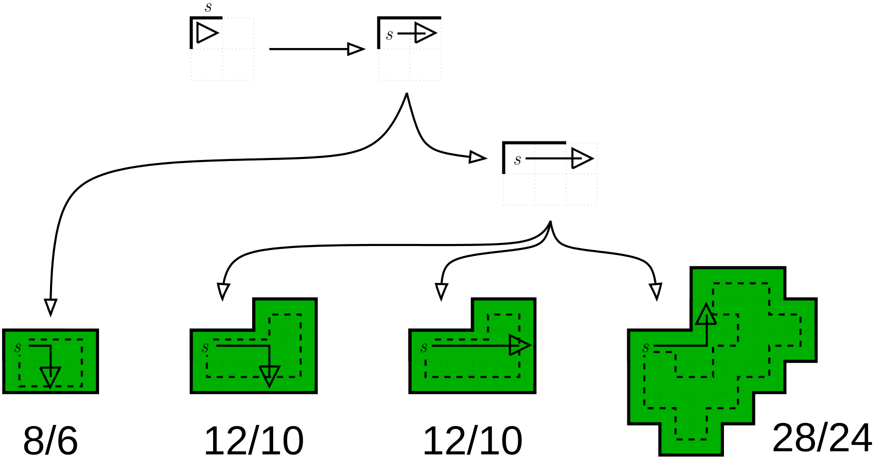

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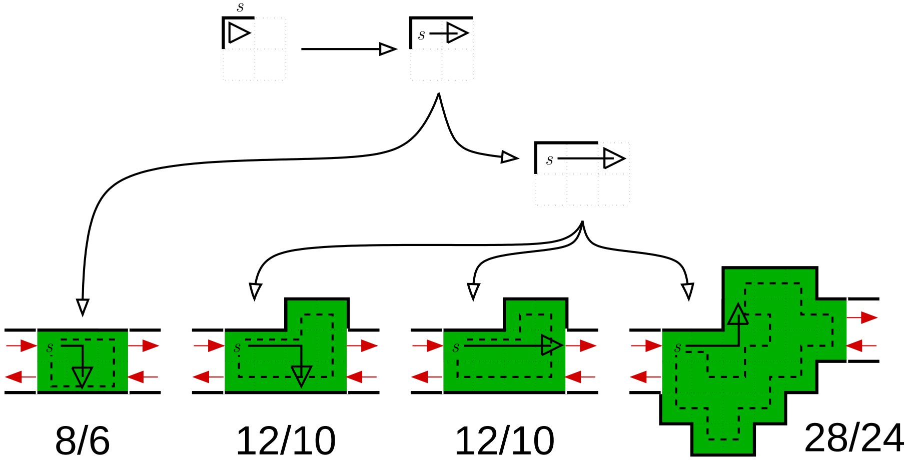
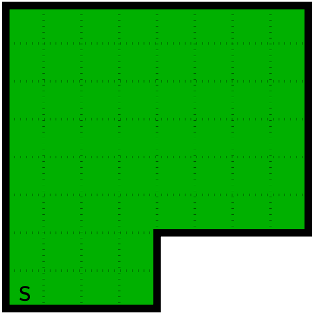
Polygons of arbitrary size



SmartDFS: An exploration strategy (1)



- First idea: Apply depth-first search (DFS)
- *Left-hand rule*: prefer step to the left over a straight step over a step to the right
- Visits *each* cell twice!

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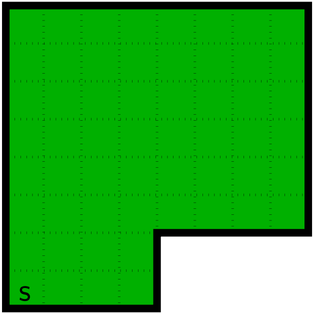

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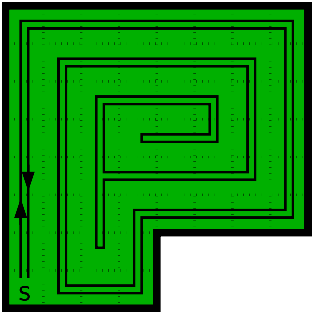

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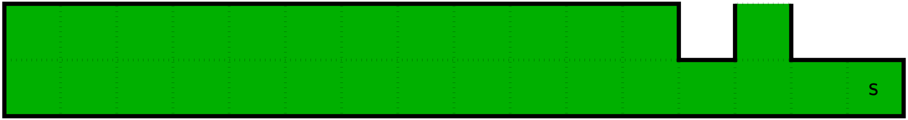

- DFS visits each cell twice
- More reasonable: Return directly to unvisited cell
- Improved DFS

Improvement 1

Return directly to those cells that have unexplored neighbors.

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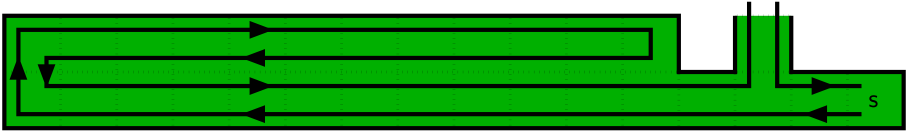

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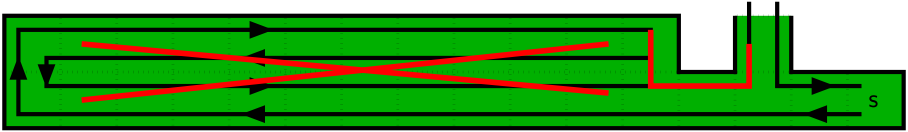

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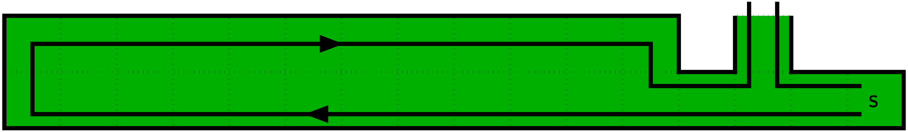

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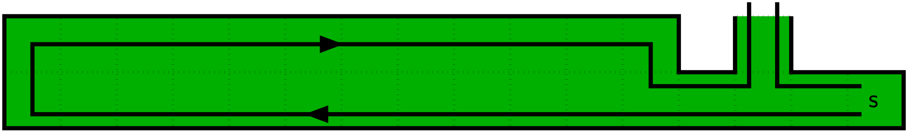

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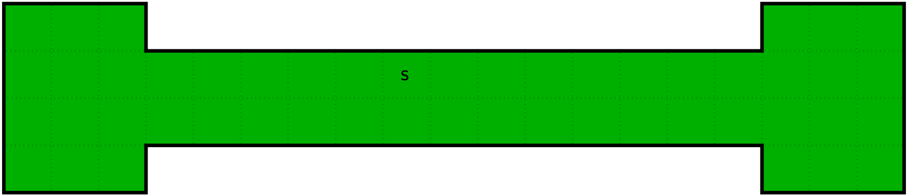

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- More reasonable: Visit right part immediately, continue with the corridor, visit left part, return to s
- Long corridor is traversed only two times!
- *Split cells*: Set of unvisited cells gets disconnected

Improvement 2

Detect and handle split cells (i. e., prefer parts of P farther away from the start).

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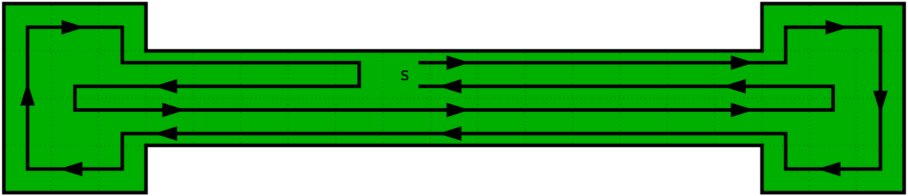

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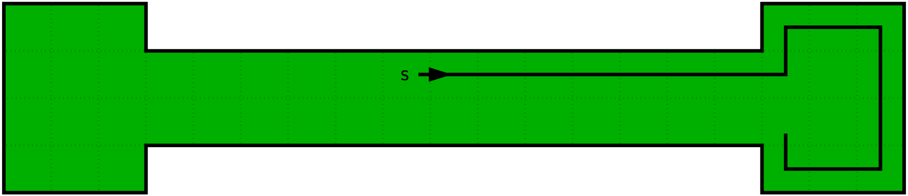

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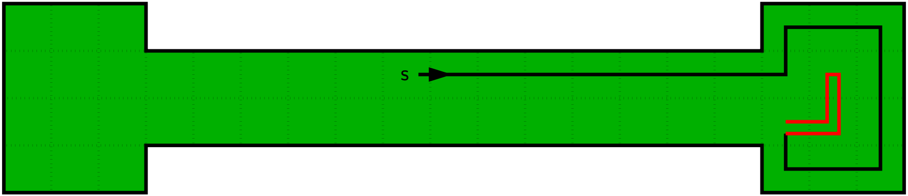

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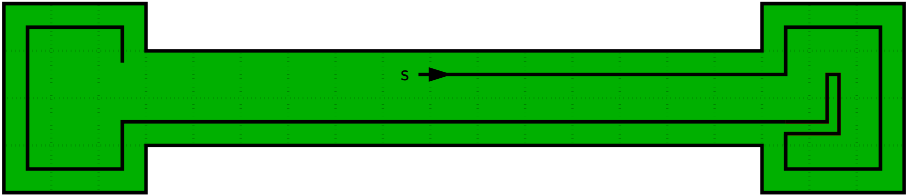

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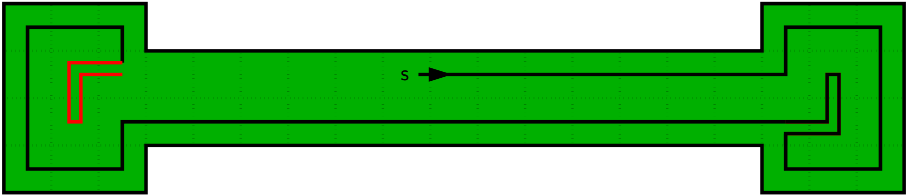

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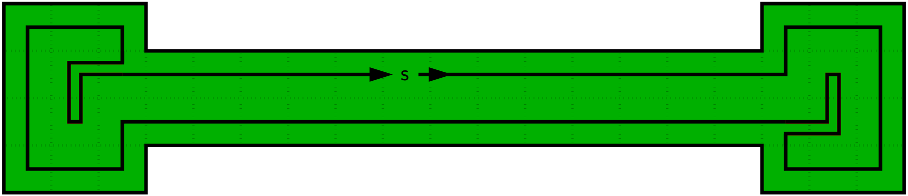

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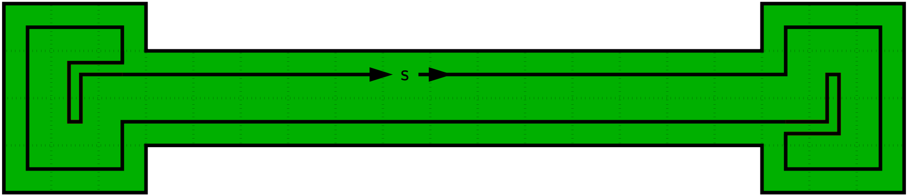

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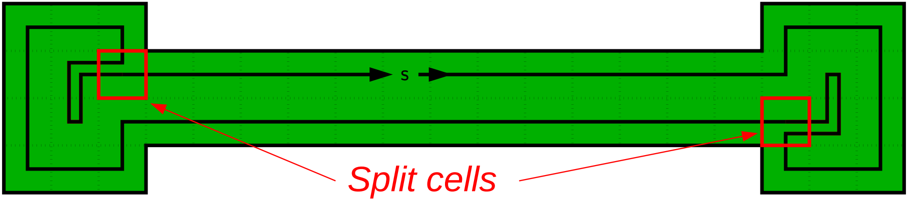

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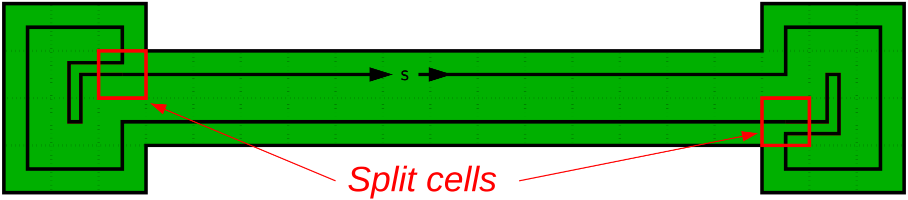

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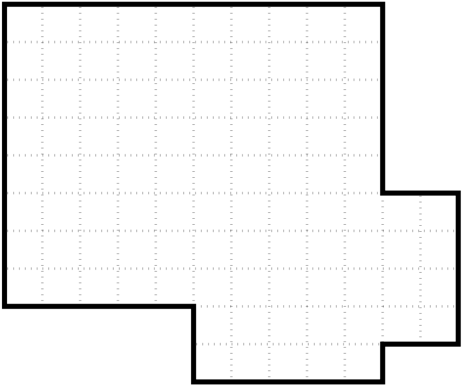
Java Applet

http://www.geometrylab.de/Gridrobot/

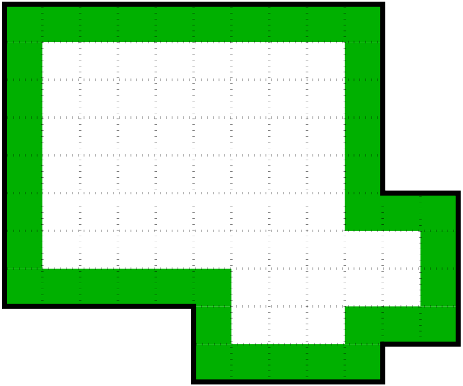
Layer and Offset



- *First layer* :=
  Boundary cells of P
- *1-offset* :=
  P without first layer
- Analogously: *Second layer*
- *2-offset*
- and so on
- E: #edges between free and blocked cells

Lemma (Number of edges)

P' *is* ℓ*-offset of* $P \Rightarrow E(P') \leq E(P) - 8\ell$*.*



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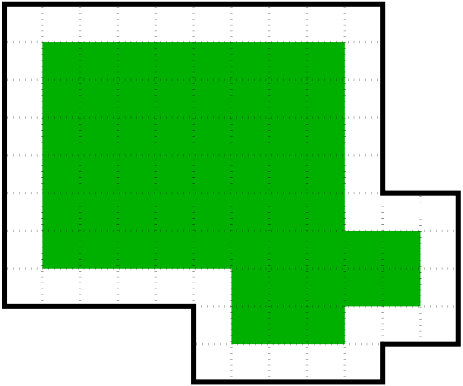

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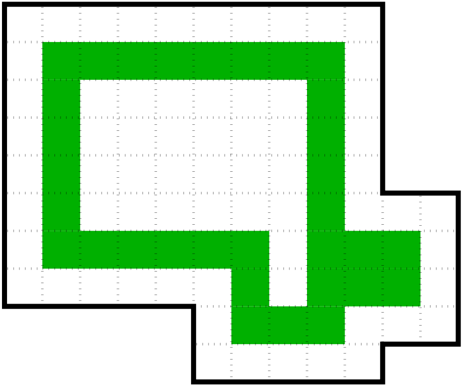

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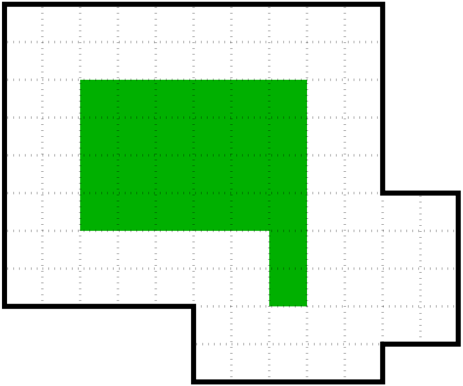

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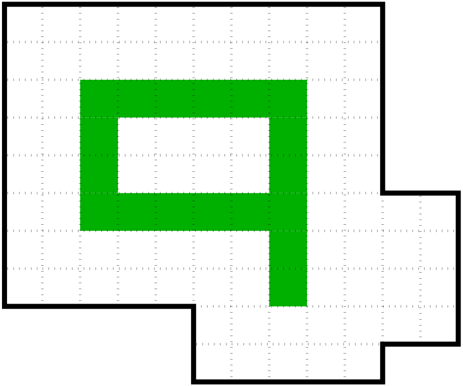

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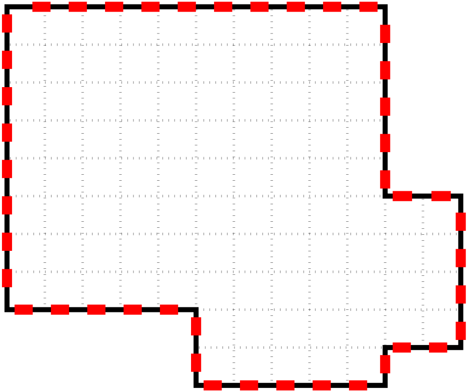

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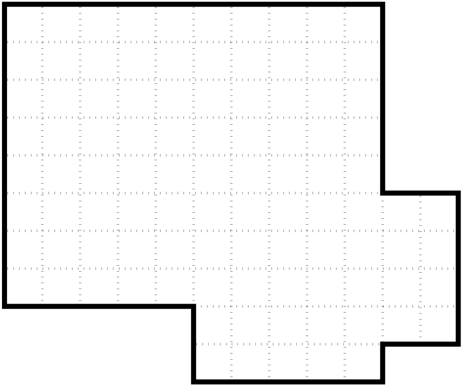

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Shortest path between two cells in $P \leq \frac{1}{2}E(P) - 2$.

Proof sketch.

- Worst case:
  Both cells in the first layer
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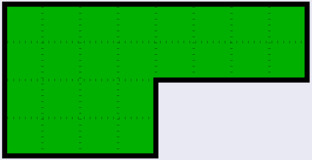

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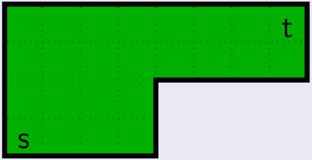

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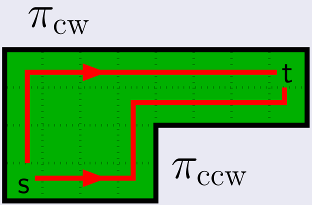

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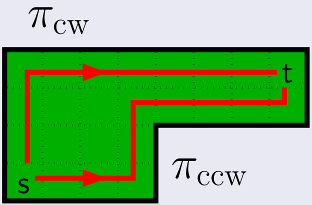

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Upper Bound on the Number of Steps

Theorem (Number of Steps)

$$S \leq C + \frac{1}{2}E - 3 \qquad \textit{(tight!)}$$

(S: #Steps from cell to cell, C: #Cells, E: #Boundary edges)

- Proof by induction on the number of split cells
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Narrow passage: Corridors of width 1 or 2.

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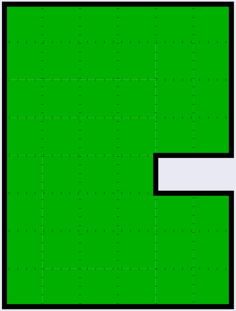

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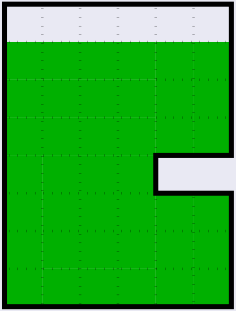

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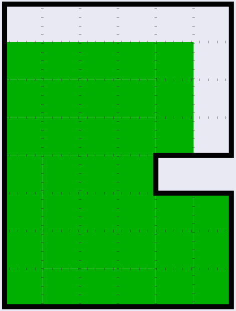

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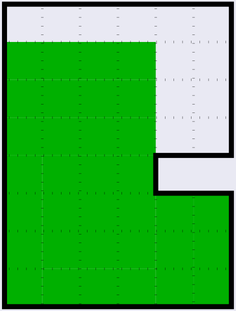

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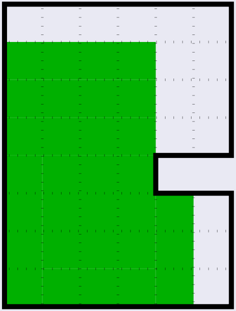

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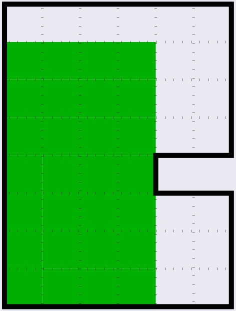

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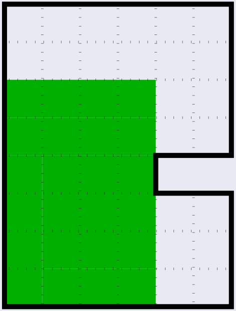

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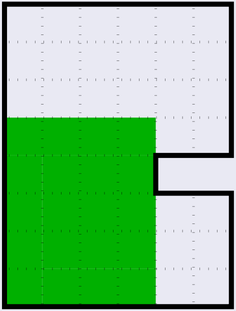

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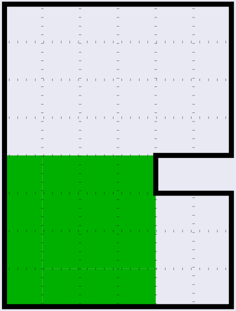

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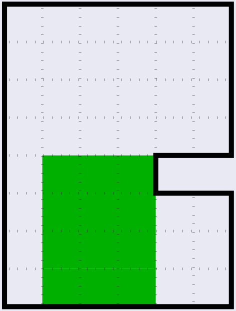

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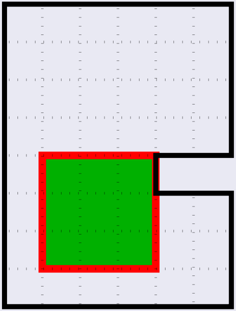

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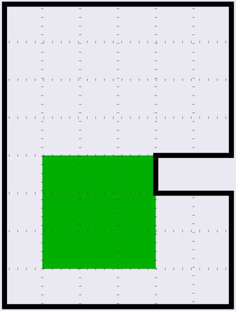

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Competitivity (3)

Lemma (Exploration of uncritical polygons)

For uncritical grid polygons: $S(P) \leq C(P) + \frac{1}{2}E(P) - 5$.

Proof sketch

- $S(P) \leq C(P) + \frac{1}{2}E(P) - 3$ shown
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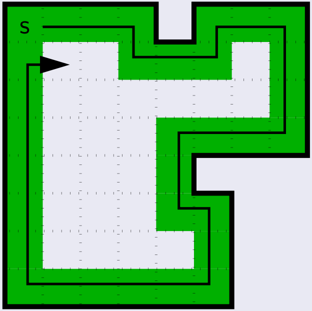

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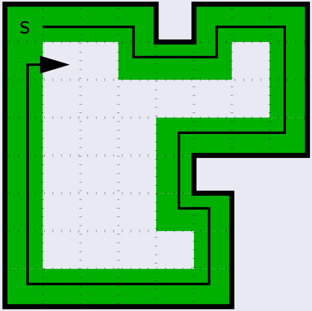

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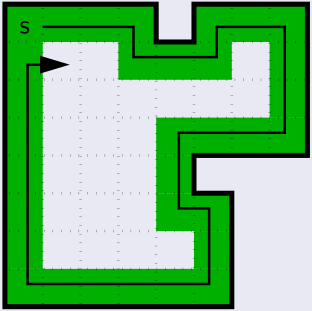

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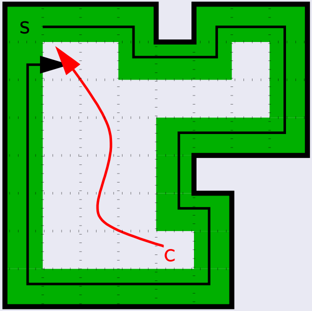

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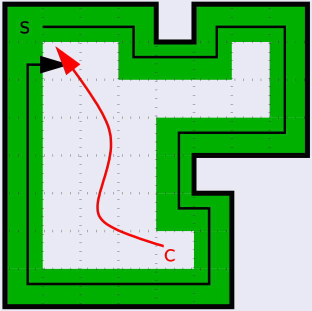

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SmartDFS is $\frac{4}{3}$ competitive.

Proof

- Remove narrow passages (explored optimally)
- $\Rightarrow$ Split P into P_i
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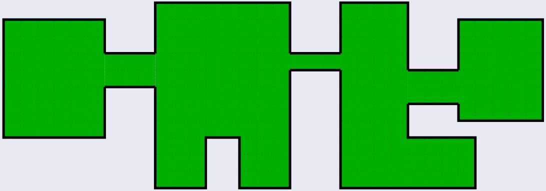

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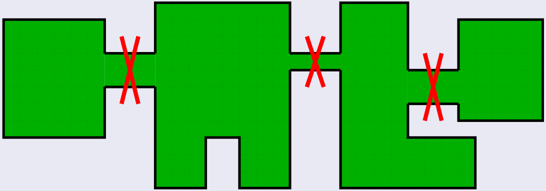

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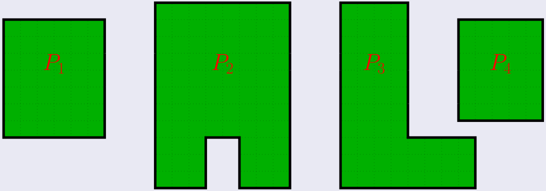

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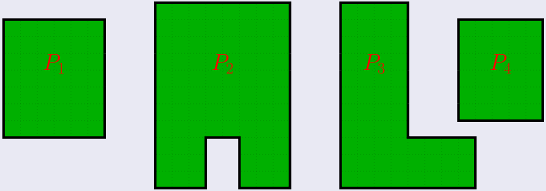

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- Show $S(P_i) \leq \frac{4}{3}C(P_i) - 2$
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$$\begin{aligned} S(P_i) &\leq C(P_i) + \frac{1}{2}E(P_i) - 5 && \text{by exploration lemma} \\ &\leq C(P_i) + \frac{1}{2}\left(\frac{2}{3}C(P_i) + 6\right) - 5 && \text{by edges lemma} \\ &= \frac{4}{3}C(P_i) - 2 \end{aligned}$$

Competitivity Proof (2)

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$$\begin{aligned} S(P_i) &\leq C(P_i) + \frac{1}{2}E(P_i) - 5 && \text{by exploration lemma} \\ &\leq C(P_i) + \frac{1}{2}\left(\frac{2}{3}C(P_i) + 6\right) - 5 && \text{by edges lemma} \\ &= \frac{4}{3}C(P_i) - 2 \end{aligned}$$

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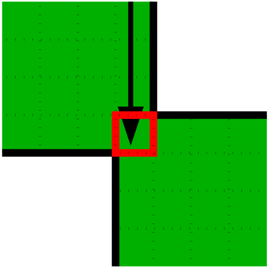

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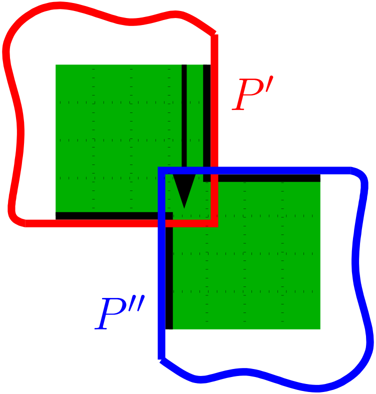

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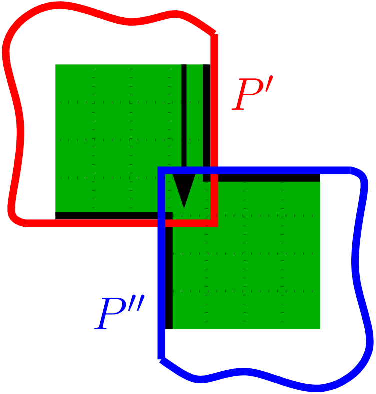

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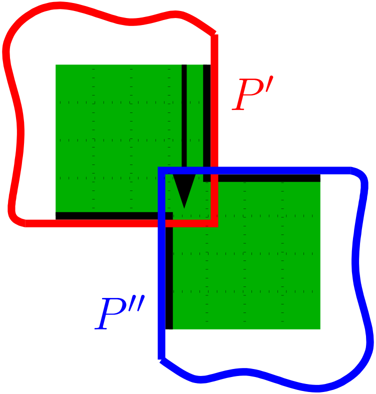

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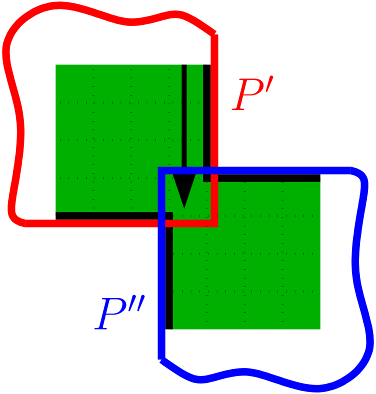

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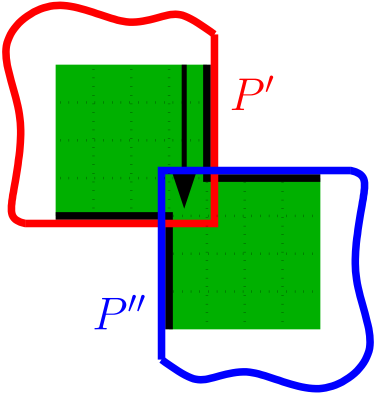

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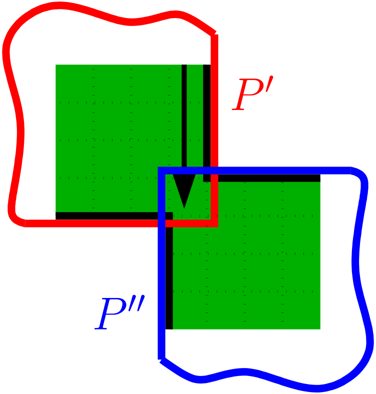

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Competitivity Proof (3)

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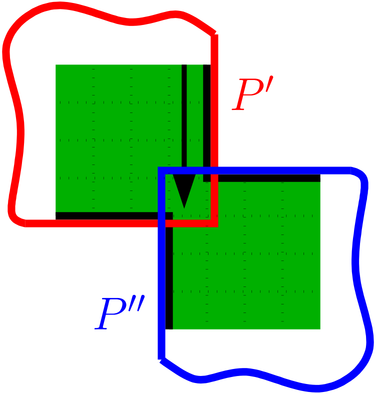

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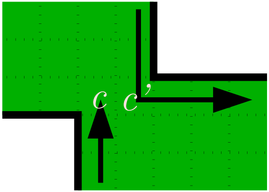
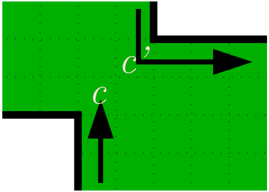
Ind. step, case 2: Robot meets cell c' touching split cell c





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Competitivity Proof (4)

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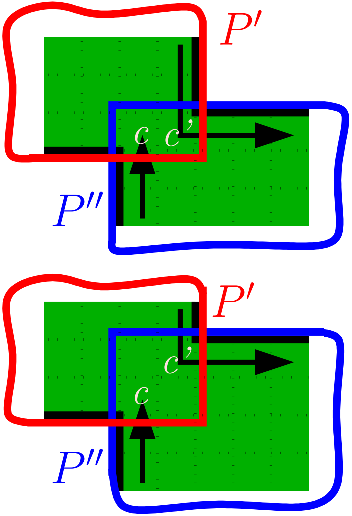

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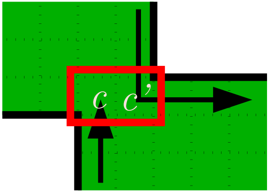

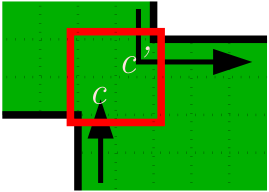

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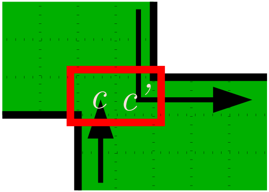

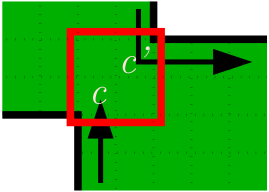

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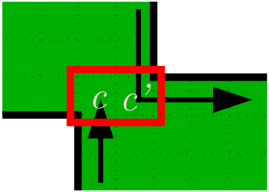

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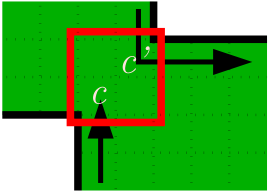

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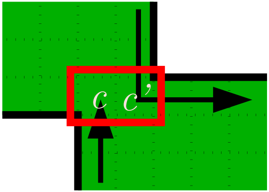

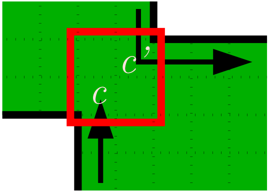

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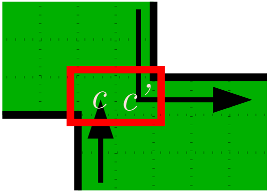

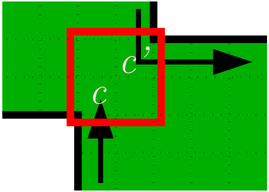

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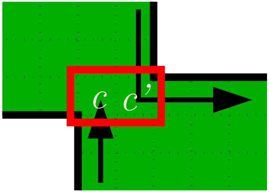

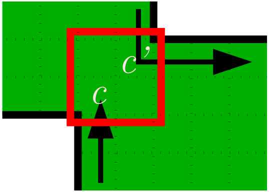

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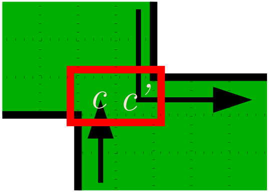

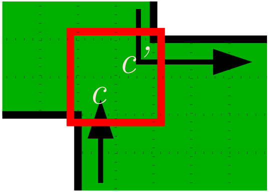

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- Lower Bound: $\frac{7}{6}$
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- $\frac{4}{3}$-competitive

- ToDo: Close the gap!

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Thank you!