Robot Swarms for Exploration and Triangulation of Unknown Environments

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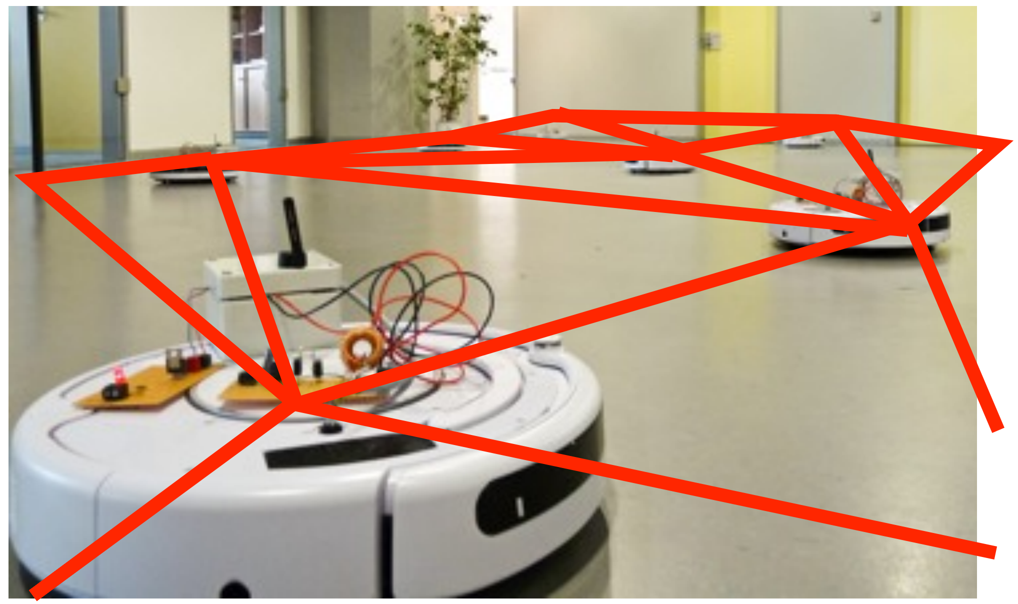
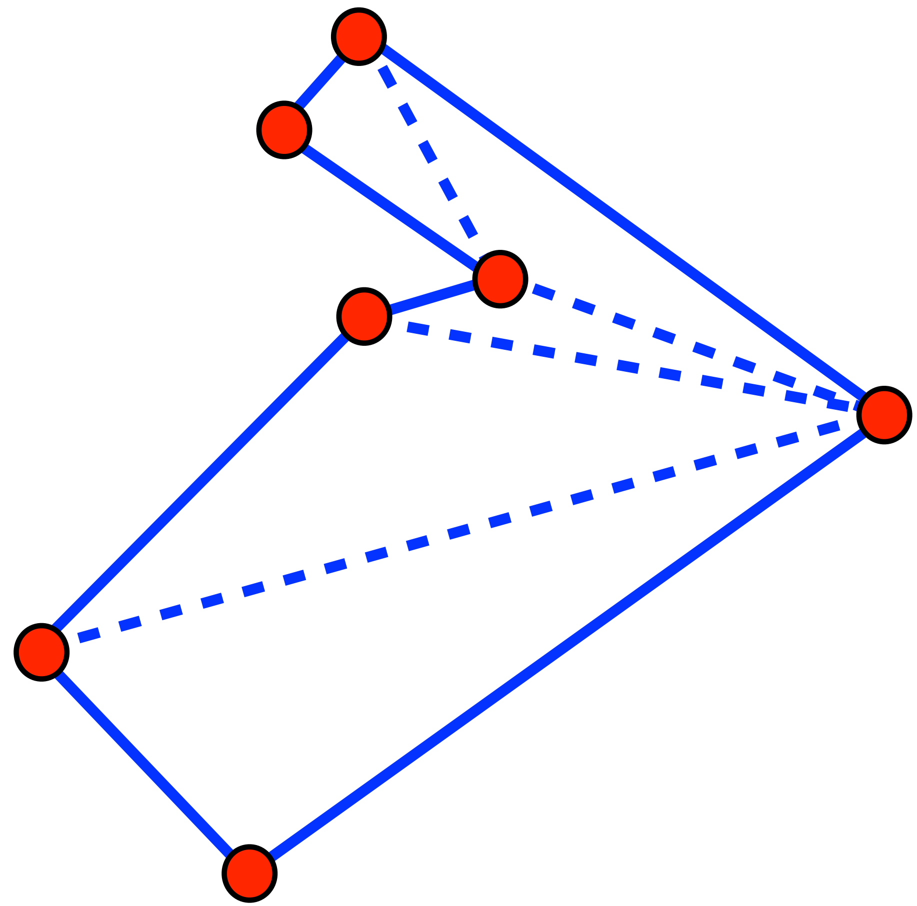
Problem Description

- Given:
  - Mobile agents
    e.g. swarm of robots
  - Unknown environment
- Goal:
  - Triangulate environment



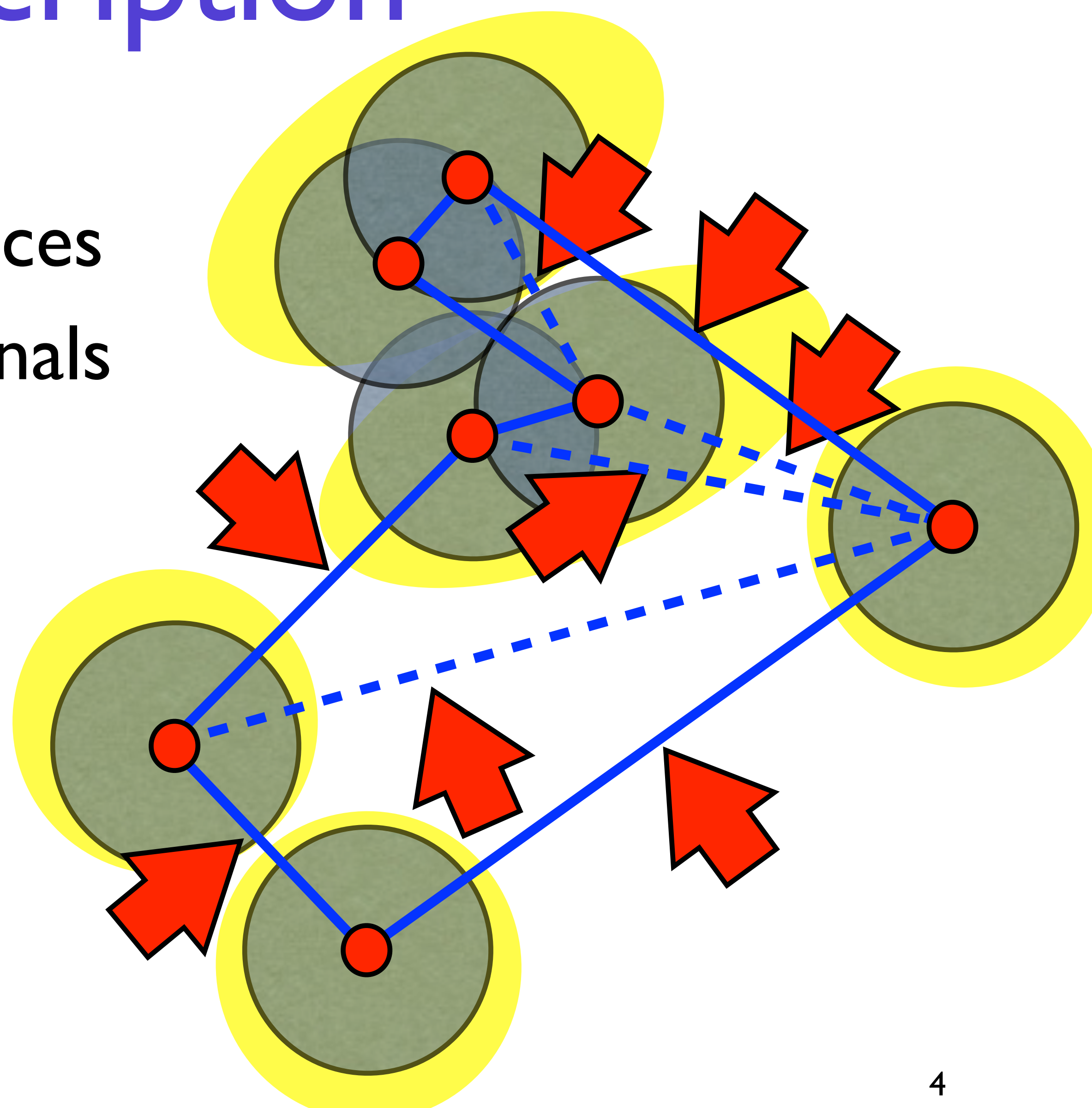
Problem Description

- Triangulation:
  - Place agents at vertices
  - Connect with diagonals



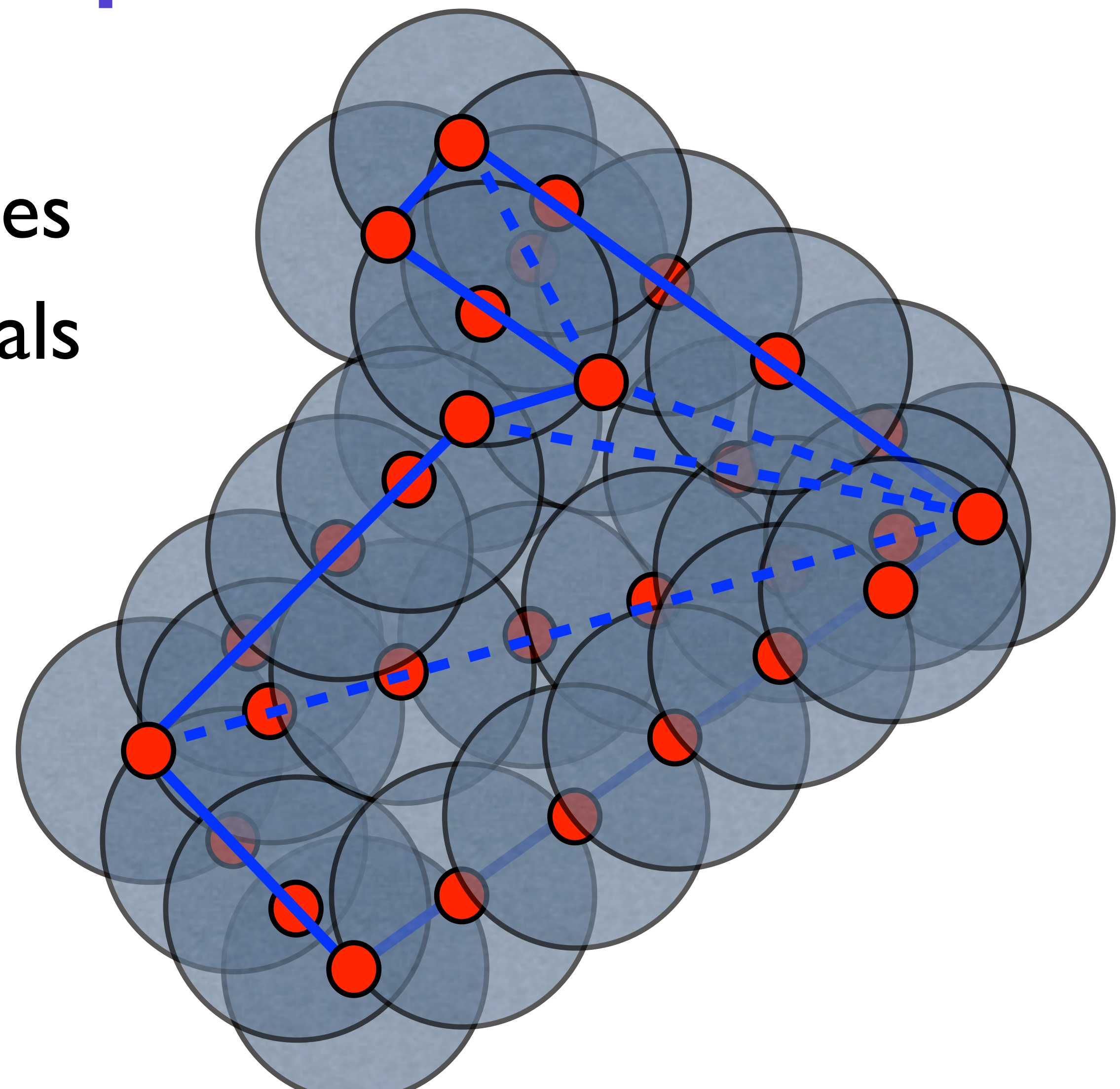
Problem Description

- Triangulation:
  - Place agents at vertices
  - Connect with diagonals
- New challenge: Limited range
  - Diagonal > range
  - No connection!



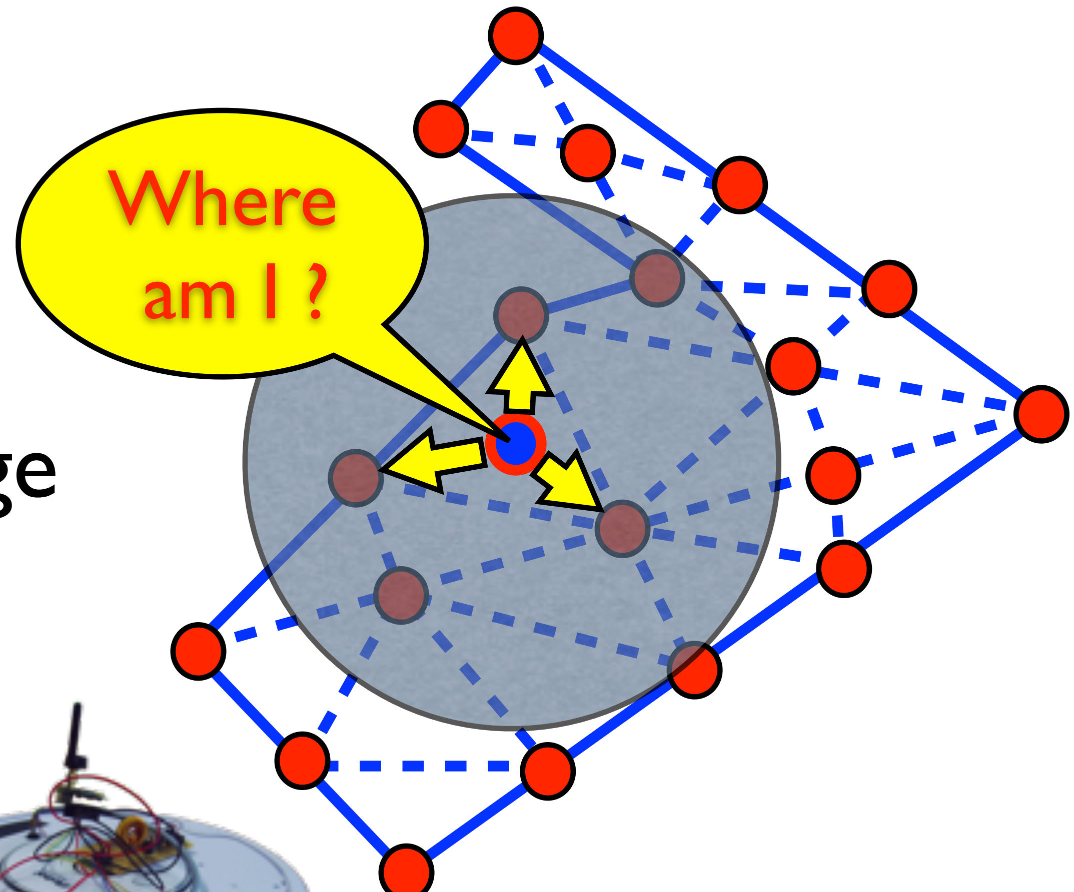
Problem Description

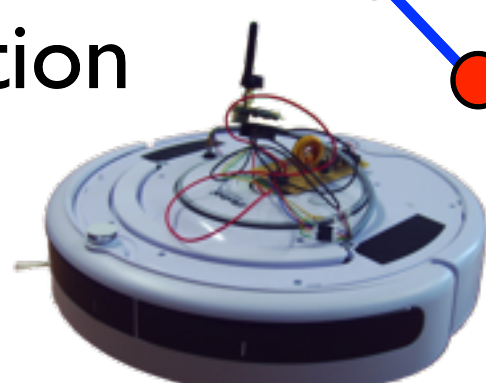
- Triangulation:
  - Place agents at vertices
  - Connect with diagonals
- New challenge: Limited range
  - Diagonal > range
  - No connection!
  - Additional agents needed



Applications

- Place guards with limited vision
- Place relays with limited comm. range
- Place beacons for robot localization





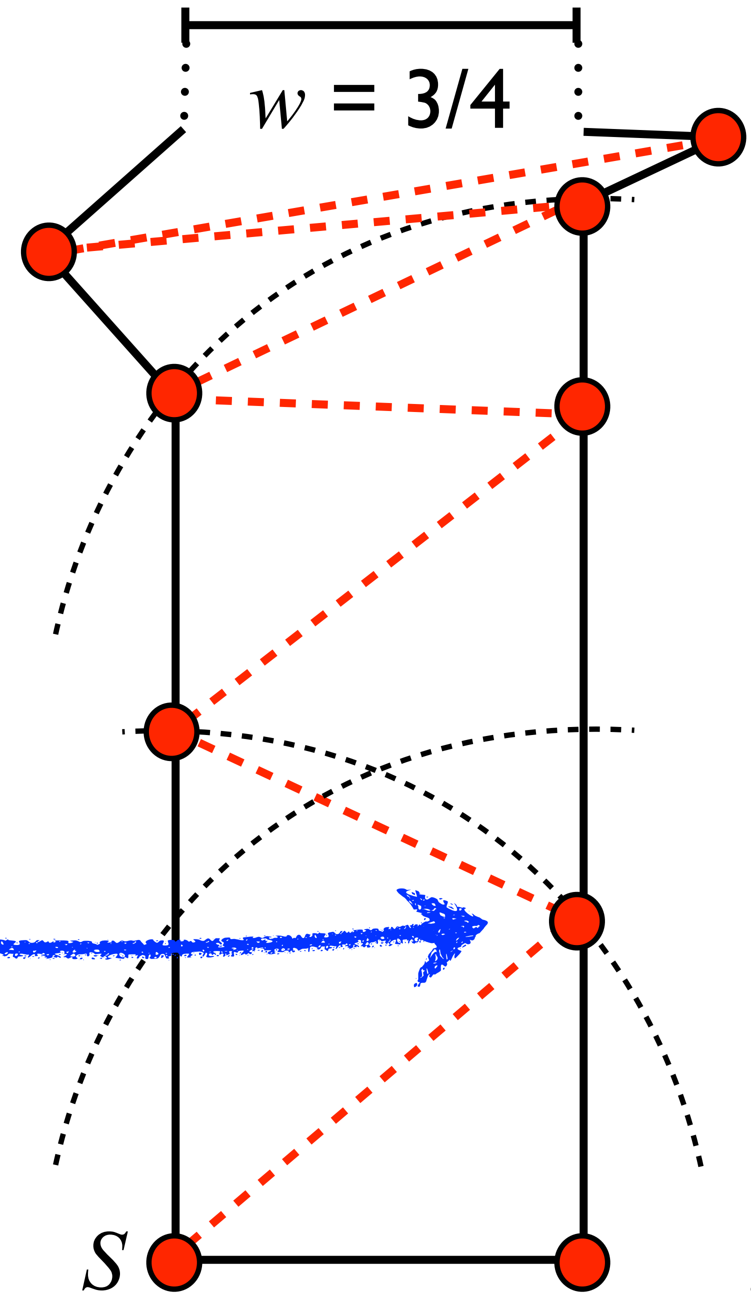
More Formally

- Given:
  - Unknown Polygon P
  - Mobile agents
    - Communication range r (r = 1)
  - Start point S on ∂P
- Task:
  - Triangulate P
  - No diagonal > 1
  - Minimize number of agents

Boundary of P

Lower Bound

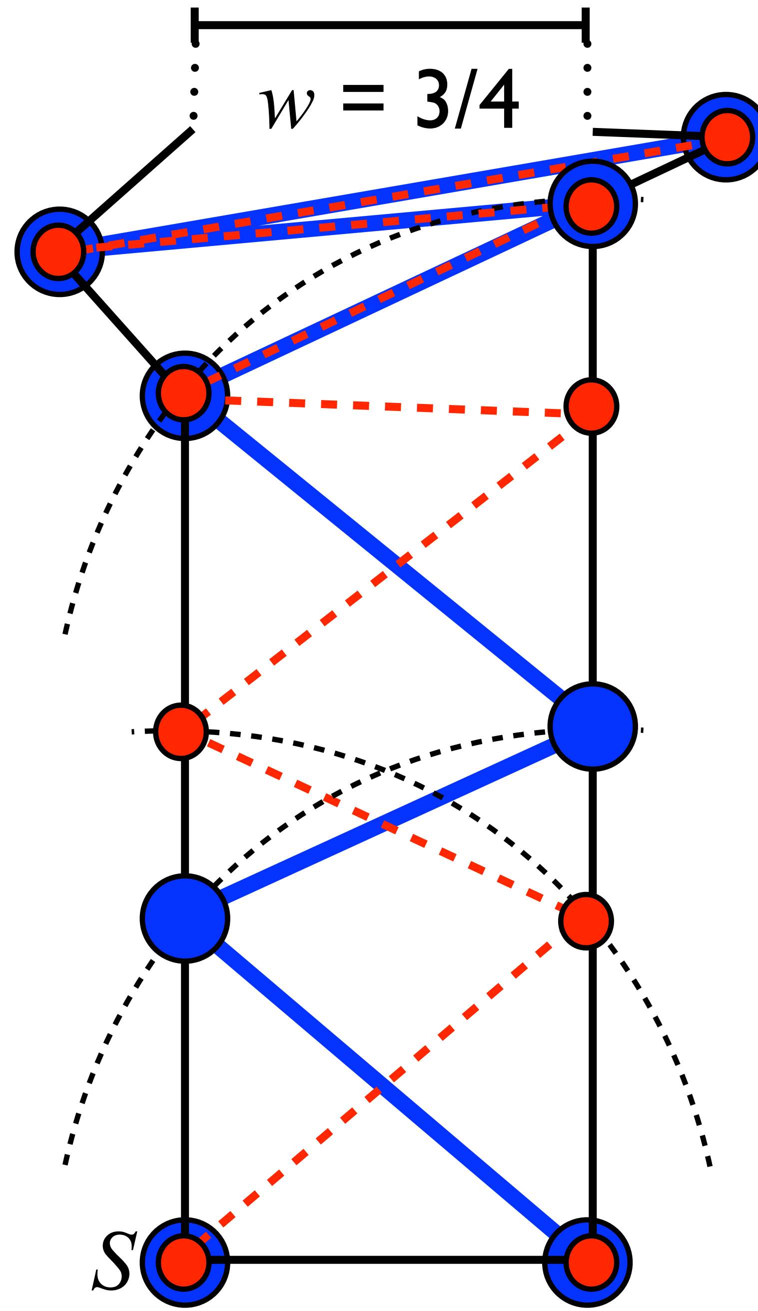
- Corridor of width 3/4
- First 2 relays at the vertices
- Case I:
  Third relay on the boundary
- 9 relays



Lower Bound

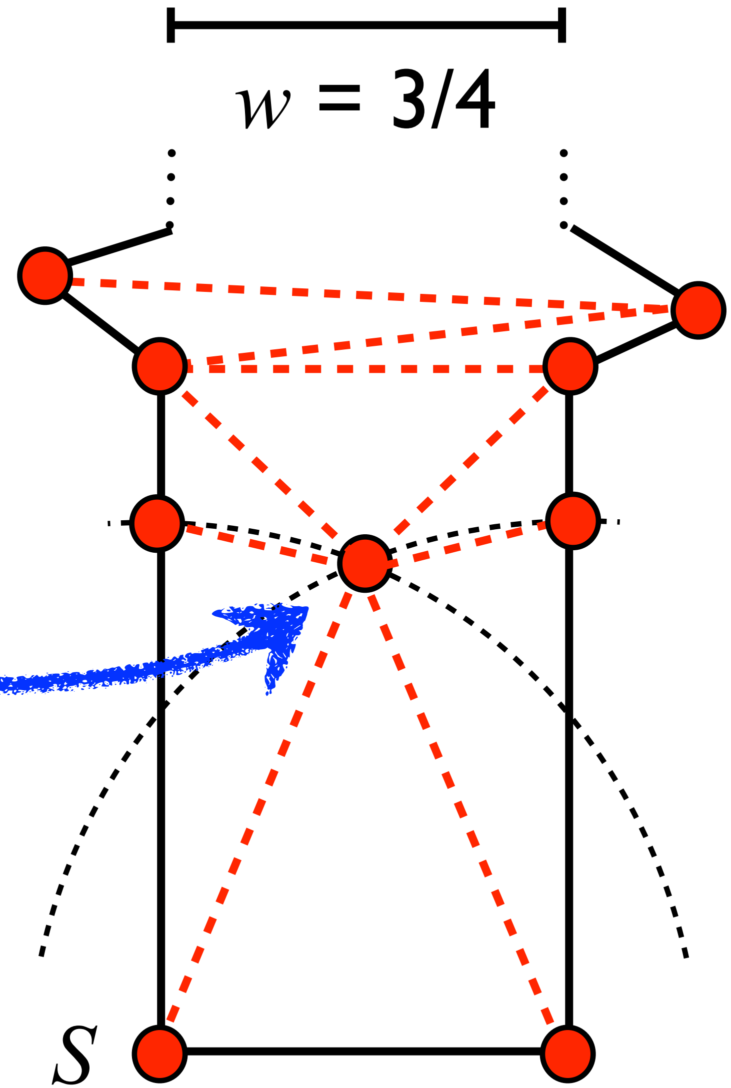
- Corridor of width 3/4
- First 2 relays at the vertices
- Case I:
  Third relay on the boundary
- 9 relays
- Optimal: 8 relays

$$\frac{\text{Alg}}{\text{Opt}} \geq \frac{9}{8}$$



Lower Bound

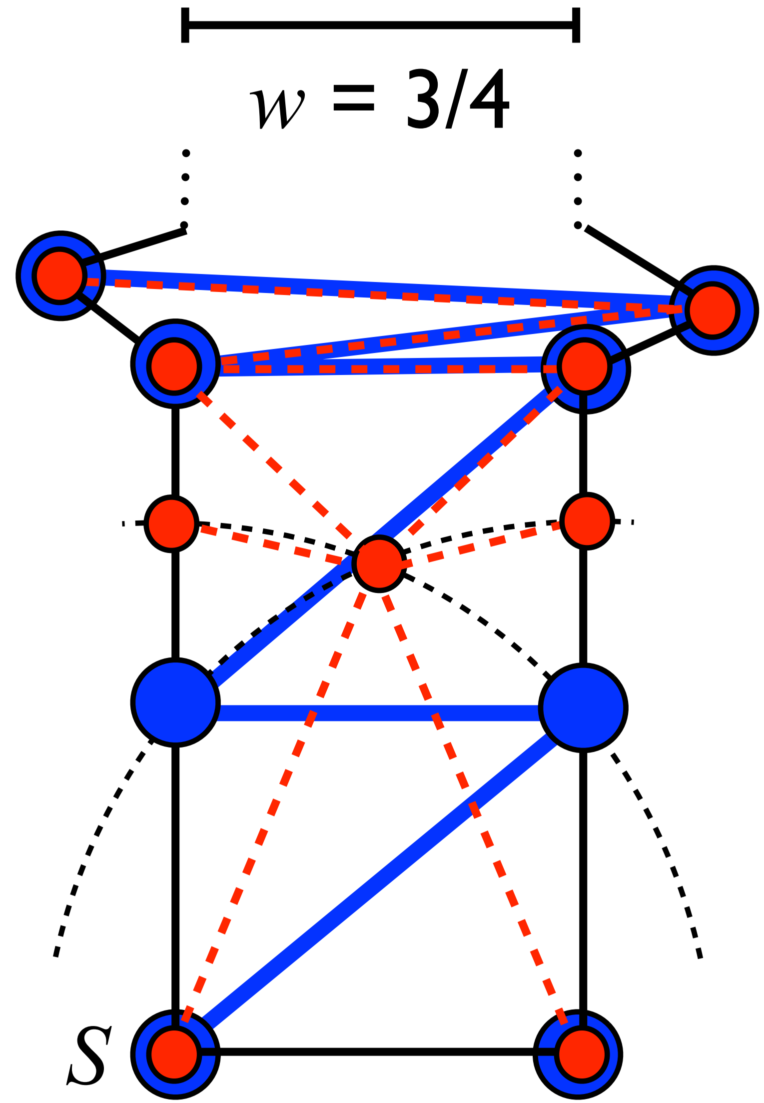
- Case II:
  Third relay in the interior
- 9 relays



Lower Bound

- Case II:
  Third relay in the interior
- 9 relays
- Optimal: 8 relays

$$\frac{\text{Alg}}{\text{Opt}} \geq \frac{9}{8}$$



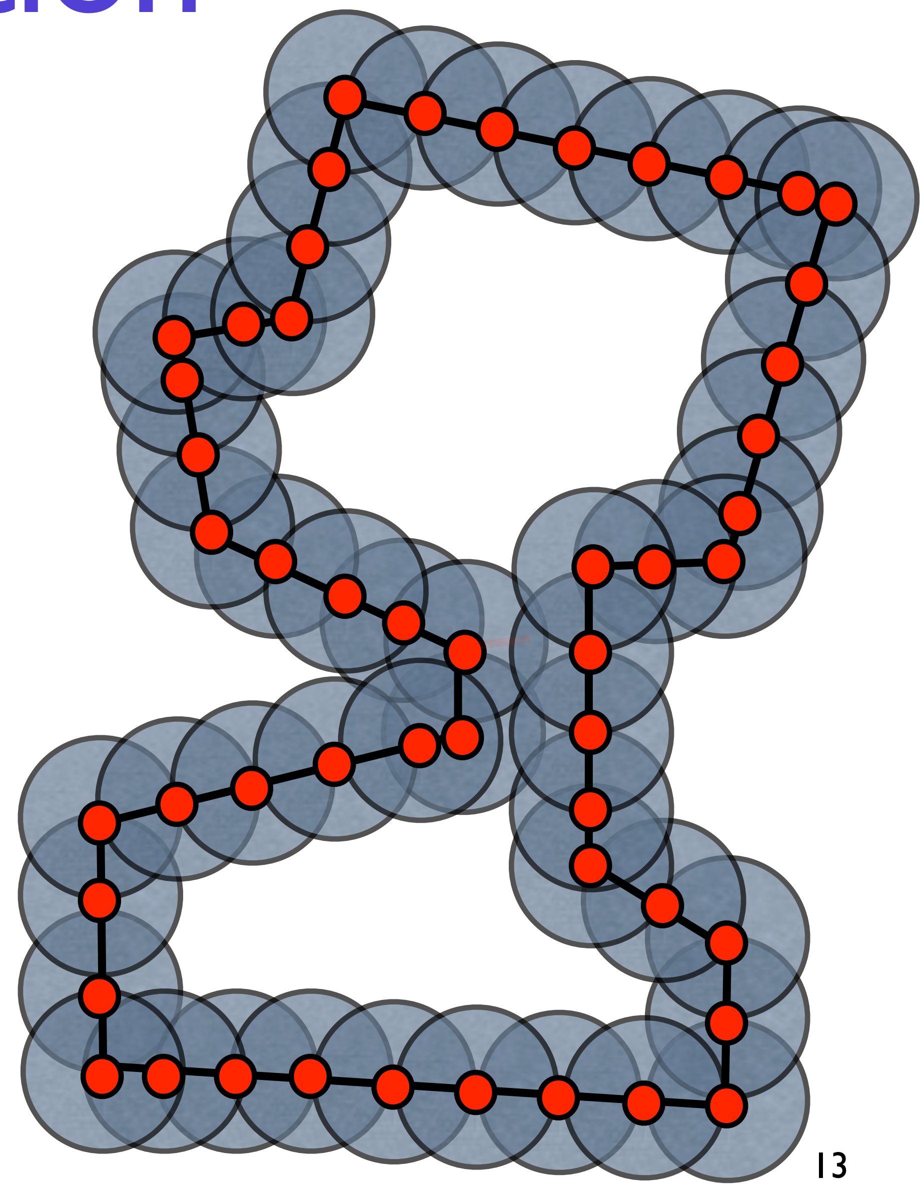
Lower Bound

Theorem

No online algorithm for the triangulation of a polygon can achieve a competitive ratio better than 9/8.

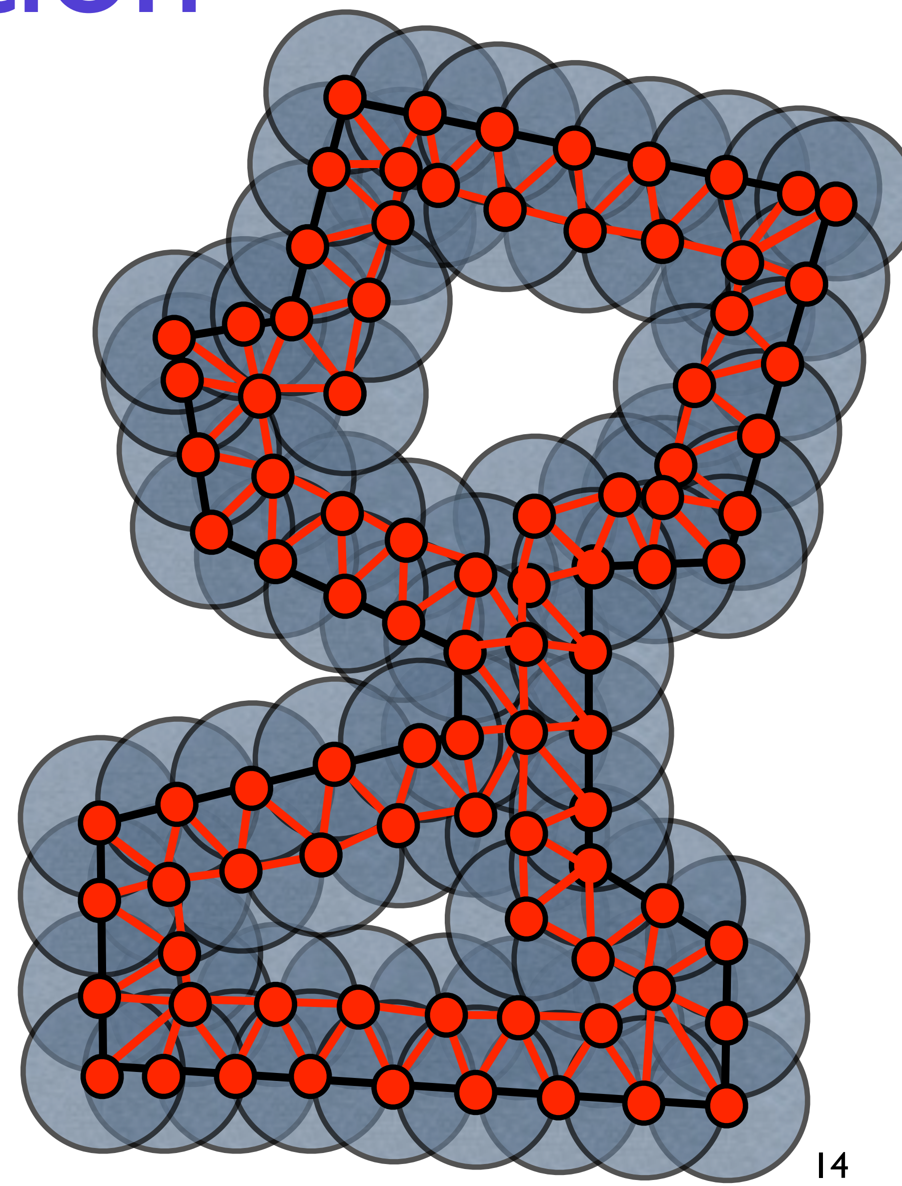
Online Triangulation

- Boundary:
    - Place relays at distance ≤ 1 on ∂P



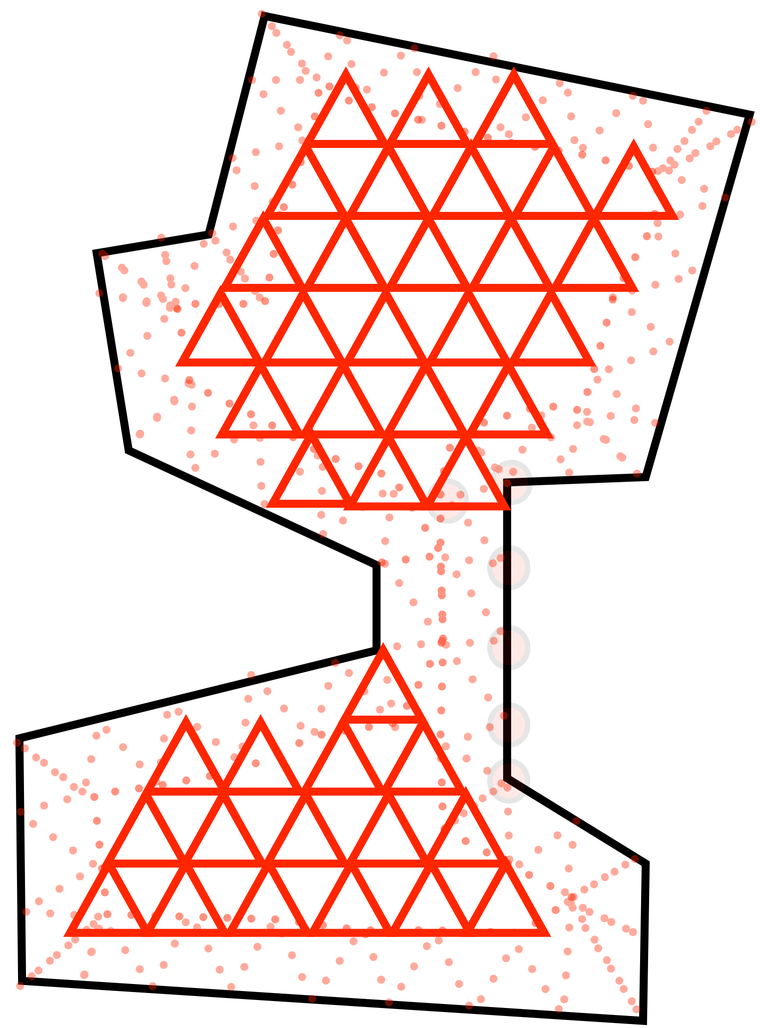
Online Triangulation

- Boundary:
  - Place relays at distance ≤1 on ∂P
  - Place second layer at distance $\leq \frac{\sqrt{3}}{2}$



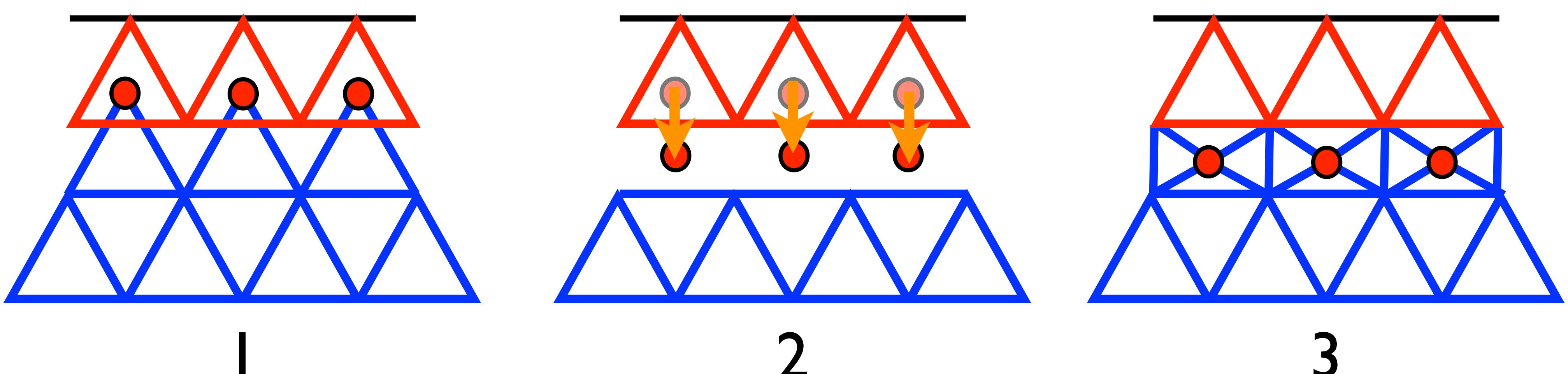
Online Triangulation

- Boundary:
  - Place relays at distance ≤1 on ∂P
  - Place second layer at distance $\leq \frac{\sqrt{3}}{2}$
- Interior:
  - Regular grid



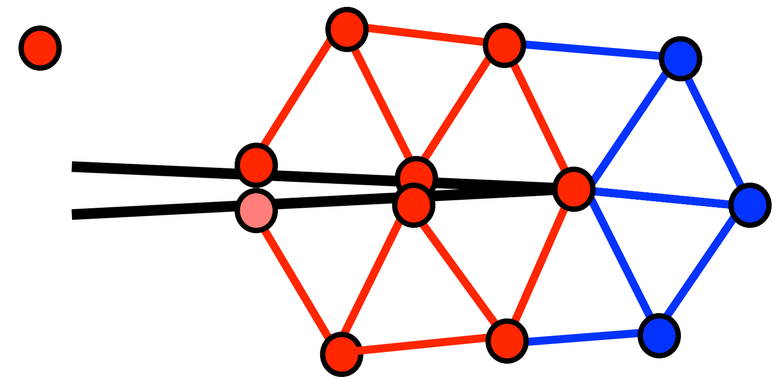
Online Triangulation

- Merge both:
  - Move relays outside the boundary layer
  - Connect



Costs: Boundary

- Boundary: 2 relays per unit
- Reflex vertex: 3 additional relays
- Convex vertex: no additional relays



$$2\,|\partial P| + 3n$$

Length of P's boundary

Number of vertices

Costs

- Interior
  k := Number of relays to „fill“ the interior
- Total: $2\,|\partial P| + 3n + k$

  ≤ 6 Opt
- Optimum needs:
  - at least $|\partial P|$ relays
  - at least k relays
  - at least n relays

Costs

Theorem

There is a 6-competitive algorithm for triangulating polygons.

Summary

Problem: Triangulation with limited range

Upper bound: 6

Lower bound: 9/8

Todo:
Narrow the gap!